Mathematical model for production sequencing in a manufacturing company

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Abstract. The objective of the research was to determine the optimal sequence of production of n jobs in m operations, in a small footwear manufacturing company with characteristics of a flow shop machine environment, which minimizes the total time of completion of jobs in the production system, makespan. A mathematical model of integer linear programming was designed, coded and solved by means of the JuliaPro software. It shows the production sequence that does not consider permutation, and minimizes makespan with 3807 minutes. The mathematical model of optimization is presented as a flexible and valuable tool for operational decision making, which could be replicated in other manufacturing companies, impacting on the improvement of costs and delivery times.

1. Introduction

The programming of operations focuses on finding the best way to use the existing capacity, considering production technical constraints; it is a type of programming (scheduling) where jobs are assigned to the machines during specific periods while satisfying one or more targets [1,2]. Pinedo [3] mentions that a sequence usually corresponds to a permutation of n jobs which must be processed in a given machine. For Mendez [4] these jobs are usually placed in Gantt diagrams.

According to the flow of products or processes, production systems are classified as continuous and intermittent. Among the intermittent are the flow shop systems and the job shop [5]. Flow shop is characterized because there are n jobs waiting to be processed in m machines, where all jobs have the same route or process sequence. For Gupta and Stafford [6] the jobs do not have to be processed in all the machines, that is, a job can skip some operations according to their technological requirement. For Buzzo and Mocellin [7] the solution of the flow shop problem seeks to optimize a measure of programming performance usually associated with time. Reza Hejazi and Saghafian [8] mention that there are usually $(n!)^m$ different alternatives for sequencing jobs in machines, however, if permutation is assumed as in most investigations, the search space is reduced to (n!), which is considered a classic problem of the flow shop.

The application of optimization models has a broad contribution in reducing costs and saving resources in all types of organizations, which gives it more relevance among the international scientific community [9]. Many researchers recognize that scheduling problems can be solved optimally using mathematical programming techniques [10]. The integer linear programming is widely used to solve real organizational problems, in areas such as production scheduling and material requirements, allowing decision making for effective business management [11].

It is necessary for companies to begin the path of continuous improvement by implementing a correct production programming system; "Unfortunately, many producers have ineffective production programming systems" [12]. One of the constant challenges of the production planning and control department is the determination of an effective production program. In small factories there is greater difficulty, due to the absence of a computational tools that facilitates rapid and effective evaluations of the different scenarios for production scheduling [13]. One of the biggest problems that arise in the management of operations of a manufacturing or services company is the sequential programming of activities of the different jobs required by the clients [14].

Many companies due to lack of knowledge, cannot improve their performance, which results in non-compliance with delivery times and product quality [2]. The planning and programming of activities in a company has a direct implication in the efficiency of any manufacturing or service system [15]. Programming is one of the most important decisions in production control systems [16], which is characterized by being one of the most complicated activities in the management of production systems, since it handles different types of resources and tasks simultaneously [17]. Studies on scheduling are plausible, since the operational phase may be the least treated at the academic level [4]. Scheduling is one of the main tools to improve performance in a manufacturing or services industry [18].

Among the objectives of the programming of the work center are: meet the deadlines, minimize the time of delay and the preparation times, and inventory in process, as well as maximize the use of machines and manpower [19]. Production programming techniques can result in significant improvements in utilization, idle times and makespan reduction and lateness in the workplace [2].

The Colombian footwear industry has shown growth in recent years. Its share in the gross domestic product (GDP) has been 0.27% of the national GDP and 2.17% in the manufacturing GDP, and the generation of approximately 100,000 jobs, which is valuable for the country's economy [20]. For the Colombian industrial association of footwear, leather and its manufactures ACICAM [21] footwear production is expected to grow around 6% for 2017, marked by an exchange rate, greater orders by the official sector and for the effect of rigorous customs control on imports. It also considers that considers that Colombia will be able to increase external sales of footwear and leather goods in places such as Chile, Peru, Ecuador, Mexico and Costa Rica. To face this new scenario, more efficient production processes must be implemented, and the internal processes of the footwear companies optimized, so that they become more productive and they reduce the orders delivery time.

The purpose of the research was to determine the best sequence of job in operations, in a small footwear company under a flow shop production environment, using an optimization model, in order to minimize the total time of completion of jobs in the production system (makespan).

2. Optimization method

The production programming problem corresponds to Fm//Cmax, where the machine environment corresponds to flow shop (Fm), and the objective is to minimize completion times of jobs in the system (makespan - Cmax). To solve the problem in the case study, integer linear programming (ILP) was used.

2.1. Operations research and integer linear programming

The main phases of the implementation of operations research include: 1) the definition of the problem; 2) the construction of the model; 3) the solution of the model; 4) validation of the model and implementation of the solution. In the opinion of the author, the most important technique of operations research is linear programming. All the operations research models, including linear programming, consist of three basic components: 1) the decision variables that are to be determined; 2) the objective (the goal) that needs to be optimized (maximize or minimize), and 3) the constraints that the solution must satisfy [22]. To solve the mathematical model of ILP, the JuliaPro software version 0.6.2.1 was used.

3. A case of application: small footwear company

3.1. Production process
The study unit was a small footwear company located in the city of San José de Cúcuta, Colombia, whose manufacture is making to order. The production process is composed of the following stages: for the design of the product, market trends are consulted in specialized magazines, designs launched by the competitor firms in footwear fairs around the world, and by direct observation of its target market; the cutting process is carried out manually, where the tracing of the mid-spanner is first performed, then the pieces are cut, the reinforcement and embossing lines are demarcated, and then they are classified by size and color; in the roughing one diminishes the caliber of the leather to facilitate the ensemble and sewing of the shoe, and in the embossed one it is printed in the leather the logo of footwear of the company; lining draws the seams over the pieces previously cut according to the model; in the weaving operation, a luxury seam is manually made on the shoe; in the assembled set for the template, it is covered and the assembly of the cut is carried out on it, manually; Sole fitting joins the sole to the shoe manually, with the help of an oven and a press for the vulcanizing process; then the product is cleaned and its conformity is reviewed, otherwise it is returned to the previous process; finally, shoes are packed. Figure 1 shows the productive flow of the small footwear company.

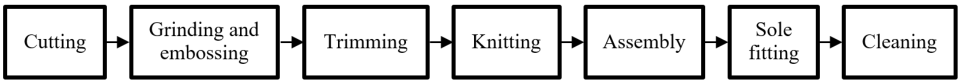

Figure 1. Small footwear company productive flow.

3.2. Description of the case study
Sequential models have applications mainly in a task workshop where a set of machines, of general purpose, execute a series of operations on job orders or production tasks. The tasks are often unique and ordered by a specific client [23]. In this investigation it was considered to determine the best sequence of production to manufacture six references of footwear through the application an optimization model, within a line of production conformed by seven operations. In Table 1, required process times (minutes/lot) are presented for each job in each process or operation.

Table 1. Processing time required for each job on each machine.

	Process or operation (j) Process time (minutes/lot)						
	1	2	3	4	5	6	7
Job (i)	Cut	Grinding and embossing	Trimming	Knitting	Set	Sole fitting	Cleaning
1	243	128	721	0	489	772	427
2	100	51	267	0	198	306	158
3	226	154	502	202	382	605	358
4	77	40	160	0	151	239	130
5	71	31	135	0	121	190	91
6	166	108	329	379	274	431	244

3.3. Assumptions of job sequencing problem jobs in the production system
In this case of study, the following assumptions were considered: 1) there is a finite set of n jobs to process in m sequential machines (operations), which are available to start their process at time zero; 2) the completion time of the jobs include preparation and processing times, and these are considered as deterministic quantities; 3) once the job has started on a machine (operation), it cannot be interrupted; 4) machines or workers are going to be available continuously, and 5) each of the machines or work stations develops a single process or operation.

3.4. Solution for integer linear programming optimization algorithm
Considering the phases of operations research, its application for the sequencing of the production in the small footwear company is shown below.

3.4.1. Problem definition. The problem was to determine the sequencing of the production for a small footwear company, specifying the time of completion for six jobs in seven operations, to optimize the total completion time of jobs (makespan).

3.4.2. Integer linear programming mathematical model. According to the basic components of linear programming, the results of its application in the case study are presented below.

3.4.3. Definition of sets. The problem identifies the set of jobs and the set of operations, the sub-indices of which are:

- i: Sub index that identifies the job, where i = 1,…, n
- j: Sub index that identifies operation j, where j = 1,…, m
- k: Sub index that identifies the job that follows job i, where k = i+1,…, n

3.4.4. Definition of integer linear programming decision variables. The decision variables whose values were determined by means of the solution of the mathematical model are:

- X_{ij}: Time of completion of job i in operation j.
- Y_{ijk}: Binary variable, used for overlapping restrictions, where: $Y_{ijk} = 1$, if job i is done before job k in operation j; and $Y_{ijk} = 0$, if job k is done before job i in operation j.
- Cmax: Auxiliary variable used to minimize the maximum completion time of jobs i in the last operation.

3.4.5. Definition of integer linear programming constant parameters. Constant parameters are:

- P_{ij}: Processing time of job i in operation j.
- P_{kj}: Processing time of job k in operation j.
- M: A large number, which must be at least the process time for job i.

3.4.6. Objective Function. The objective was to determine a production program that specifies the sequence and the completion time of each job i in each operation j, which minimizes the program termination time, makespan or Cmax, Equation (1).

$$\mathrm{Min} = \mathrm{Cmax} \tag{1}$$

3.4.7. Problem restrictions.

3.4.7.1. Minimum Cmax restrictions. They are used to find the minimum value of the maximum time of completion of the jobs in the last operation, which is denoted as m, Equation (2).

$$X_{ij} \leq \mathrm{Cmax} \qquad \forall i \in \{1, \dots, n\}, j = m \tag{2}$$

3.4.7.2. Overlapping restrictions. This disjunctive constraint prevents two jobs from being scheduled at the same time in the same operation, that is, since there is only one resource, it can only attend one job at a time. The Equation (3) indicates that the job i is performed after the job k, and Equation (4) indicates that the job k is done after job i.

$$X_{ij} - X_{kj} \geq P_{ij} - MY_{ijk} \quad \forall i \in \{1, \dots, n\}, j \in \{1, \dots, m\}, k \in \{i+1, \dots, n\} \quad (3)$$

$$X_{kj} - X_{ij} \geq P_{kj} - M(1 - y_{ijk}) \quad \forall i \in \{1, \dots, n\}, j \in \{1, \dots, m\}, k \in \{i+1, \dots, n\} \quad (4)$$

3.4.7.3. Precedence restrictions. This restriction guarantees that job i must first go through operation j-1 and then operation j, therefore, Equation (5) indicates that the start time of job i in operation j must be at least equal to the time in which operation j-1 is completed.

$$X_{ij} - X_{ij-1} \geq P_{ij} \quad \forall i \in \{1, \dots, n\}, j \in \{1, \dots, m\} \quad (5)$$

3.4.7.4. Restrictions for time of completion of jobs i in the operation 1. This restriction establishes that the completion times of the jobs i in operation 1, must be at least equal to the processing time of each job i in that operation, Equation (6).

$$X_{ij} \geq P_{ij} \quad \forall i \in \{1, \dots, n\}, j = 1 \quad (6)$$

3.4.7.5. Restriction of non-negativity. The Equation (7) impose that all model decision variables must be a non-negative integer.

$$X_{ij} \in Z^{+} \quad \forall i \in \{1, \dots, n\}, j \in \{1, \dots, m\} \quad (7)$$

3.4.7.6. Auxiliary variable restriction. The Equation (8) refers to the fact that the model auxiliary variable Cmax must be non-negative.

$$\mathrm{Cmax} \geq 0 \quad (8)$$

3.4.7.7. Binary auxiliary variable restriction. The Equation (9) impose binary values for the binary auxiliary Y_{ijk} variables of the model.

$$Y_{ijk} \in \{0,1\} \quad \forall i \in \{1, \dots, n\}, j \in \{1, \dots, m\}, k \in \{i+1, \dots, n\} \quad (9)$$

The Equation (1) to Equation (9) present the ILP mathematical model is presented, which determines the sequence and the completion time of each job, in each operation and minimizes the makespan.

4. Results

4.1. Mathematical model solution

Model code was implemented in JuliaPro software (ver.0.6.2.1). Table 2 shows the completion times of each job processed in each operation, X_{ij}, whose makespan corresponds to 3,807 minutes (63.45 hours). Figure 2 shows each job completion times and that the sequence of jobs varies for each operation.

Similarly, Habib [2] improved makespan, but by means of dispatch rules and heuristic algorithms, in a leather products factory. In the literature reviewed there is no evidence of the application of ILP to the problem of sequencing of production in the footwear production sector in Colombia, hence the relevance of this research. To schedule n jobs on m machines, through heuristic algorithms in hypothetical cases, which minimize makespan, was evidenced in [23-25], and Restrepo [26] developed an ILP to minimize penalty for late jobs. Ramírez [27] integrated the material requirements planning and the operations programming, by means of a mixed integer linear programming model, which minimizes the costs of set-up and inventory holding, in an industry. Flow shop system research has very little practical application of programming techniques, so future research should be inspired by real-life problems rather than problems found in mathematical abstractions or theoretical aspects [6,8].

Table 2. Termination times for each job in each operation.

	Termination times (minutes) - Process or operation (j)						
	1	2	3	4	5	6	7
Job (i)	Cut	Grinding and Embossing	Trimming	Knitting	Set	Sole fitting	Cleaning
1	812	1,042	1,780	NA	2,269	3,041	3,468
2	177	290	557	NA	755	1,061	1,219
3	403	557	1,059	1,261	1,643	2,269	2,627
4	77	117	277	NA	428	667	797
5	883	914	2,244	NA	2,883	3,662	3,807
6	569	1,150	2,109	2,488	2,762	3,472	3,716

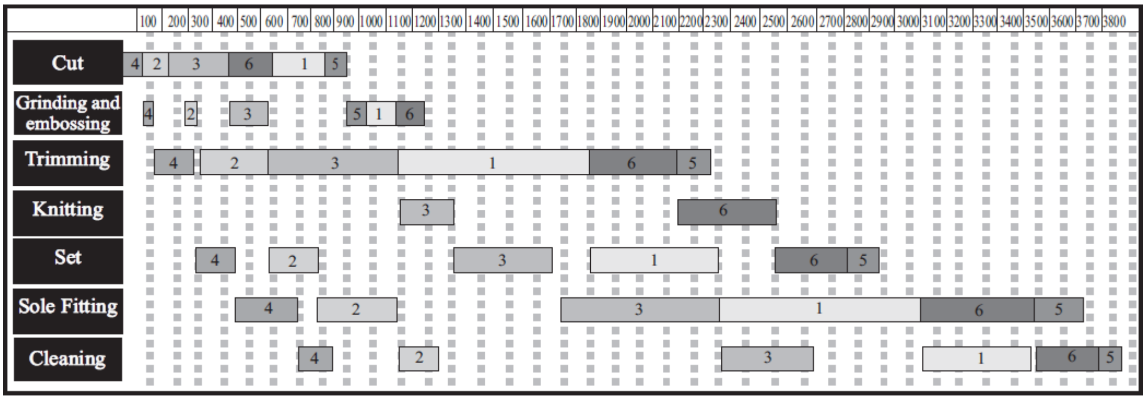

Figure 2. Sequence of production on Gantt chart.

The ILP that determines the optimal sequence of production constitutes a valuable and flexible tool, for the case study to optimally program its production, modifying the operation parameters and the sets (number of jobs and operations) for each particular situation. These results could also be applied to the companies associated to ACICAM, and to other productive sectors whose companies have similar characteristics to the case study, impacting on the improvement of their productivity and competitiveness. Similar case carried out by Habib [2] who concludes that the improvement in the leather products factory, taken as a model, is good news for the leather products manufacturing companies that are consulted by the leather industry development institute in Ethiopia, who suffer from problems of low productive performance.

5. Conclusions

The best sequence of the production of n jobs in m operations, in a small footwear manufacturing company whose productive environment is the flow shop, was determined from the solution of an integer linear programming model, which optimizes the time of completion of jobs (makespan) with a value of 3,807 minutes. The ILP, once designed and coded in the JuliaPro software, becomes a flexible and valuable tool to make decisions in production sequencing, which results in better management of the production system, strengthening competitive business strategies of speed in delivery and cost reduction. In addition, it could be replicated in any footwear manufacturing company under a flow shop productive environment, and in other productive sectors that present the same characteristics of operation of the company studied. As a future study, this problem could be solved through heuristic and metaheuristic methods, and performance measures could be compared with the optimization method.

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