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Variability of live loads computed from the analysis of internal forces produced during service life of dwellings

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Abstract. In the field of civil engineering, live loads are related to the use of the floors of buildings and dwellings. Live loads are produced by weight of people and furniture. Such loads generate internal physical effects such bending moments, shear forces or axial forces on structural elements that can show high uncertainty in magnitude and position. This work proposes the application of a physical-mathematical model which considers such uncertainty when forecasting maximum probable live loads to be used in structural analysis and design of elements. To do so, field data of live loads acting on different floors of 20 dwellings built in San José de Cúcuta, Colombia, were collected and used to verify the model validity. As a result, it was observed a small difference in forecasted live loads with respect to the live loads prescribed by the national and international structural design standards. Live loads forecasted using the model describe modern trends of use of floors and can be used as safe values for designing structural elements. Physical quantities related to bending moment effects resulted to influence importantly the obtained maximum loads values.

1. Introduction

In civil engineering, live loads are related to the weight of people and movable things which can be placed and easily removed in any place at any time upon a floor. Furniture, people, and animals can be considered as live loads [1,2]. Live loads can last seconds, days, months or years demanding a floor and according to their duration are classified as instantaneous, sustained, and extraordinary [3]. An instantaneous load has low magnitude and uses the floor for some hours or less. Also, an extraordinary load can last some hours, but it will surely have a larger magnitude. Daily use of a floor is associated to instantaneous and sustained loads while extraordinary loads are generated, for example, during a party. Typically, furniture is classified as sustained live load and presence of people is responsible for instantaneous and extraordinary live loads.

Determination of vector spaces related to internal forces such as bending moments, shear forces or axial forces on structural elements has paramount importance in forecasting maximum live loads [4]. Those vector spaces allow to forecast the magnitude, frequency of occurrence and position of live loads. Instantaneous, sustained, and extraordinary loads must be properly combined to find what is the most adverse condition that will probably demand the floor. In dwellings, the different life customs of people living in it through the time can impose an important variability to the described combinations. According to this, the described characteristics of live loads cannot be modeled using deterministic variables. The behavior of such characteristics must be analyzed and forecasted taking into consideration the spatial and temporal randomness [5,6].



In this work, a probabilistic model of live loads in dwellings which considers the spatial and temporal variability and the generated internal forces is presented and applied using field data. Real data from a sample of dwellings in San José de Cúcuta, Colombia, containing information related to weight, position, movability, and relative importance of furniture and people occupying floors were collected. As a result, mathematical expressions useful to structural engineers are proposed to be considered in the analysis and design of structures of floor in dwellings.

2. Physical–mathematical model

In this section a physical – mathematical model proposed by Peir and Cornell, 1973, and complemented by Corotis, 1979, was adapted to particular conditions of San José de Cúcuta, Colombia, [3,6]. Figure 1 shows a summary of the methodology followed to reach such objective. To compute internal forces acting in structural elements, simplified theoretical model expressions were fed with field data. Bending moment and shear forces generated at each end of beams belonging to each floor were computed using the statics of a particle principles and compatibility equations stated by the physics [4]. Live loads associated to each internal force were determined considering three percentiles, 50%, 90% and 99%, which were related to medium (service), intermediate, and maximum (design) loads values respectively. Finally, maximum loads values were adjusted using appropriate statistics that are described in this section.

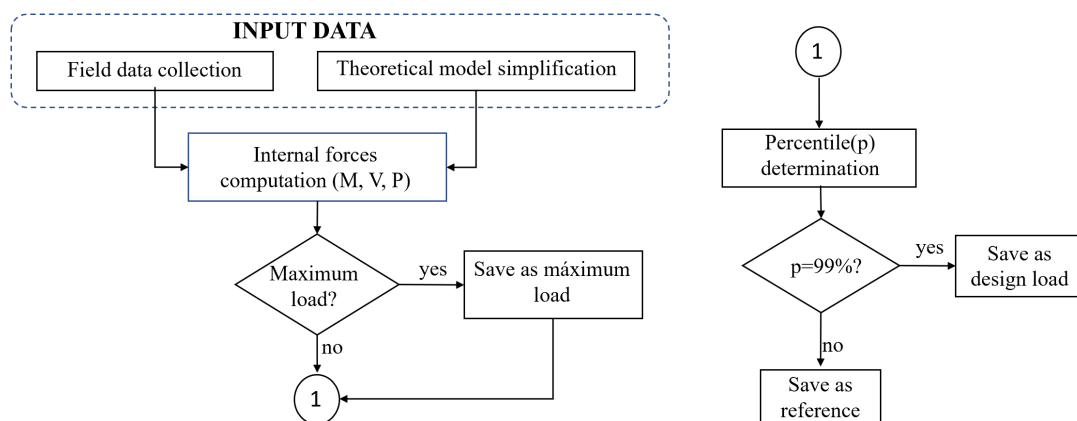


Figure 1. Flow chart of the physical–mathematical model.

To study the temporal and spatial variability of live loads, it is ideal to collect data during a large time interval. However, due to the private nature of dwellings, the continuous availability for collecting information through the time cannot be guaranteed. For this reason, to forecast their possible variability across the time it is necessary to measure the magnitude and position of live loads of a floor area at a given instant of time. In practice, the data obtained for this purpose are usually rough estimates of total weights of elements or persons, the position of their center of gravity, and the size of the area they affect. To refine the forecast of movement and magnitude of such loads upon the partial and total areas of the floors, a probabilistic model is used. Such model allows to convert discrete data into continuous random variables letting forecast how live loads act upon floors of a civil engineering constructions [3,6]. To model the sustained load intensity $w(x, y)$ a linear probabilistic model based on a load surface as that shown in Figure 2 and represented by the Equation (1) is used [6-8]:

$$w(x, y) = m + \gamma_e + \gamma_p + \varepsilon(x, y), \quad (1)$$

where m is the mean of the entire load population, γ_e the mean zero random variable used to adjust the intensity variations between dwellings, γ_p the mean zero random variable used to adjust the intensity variations between floors of the same dwelling, and $\varepsilon(x, y)$ the mean zero random variable used to adjust

the intensity variations within a tributary area of the same floor (Figure 2). In Equation (1), $\epsilon(x,y)$ describes the stochastic loading process which is initially considered to be correlated process. In a typical floor, only $\epsilon(x,y)$ varies within $w(x,y)$.

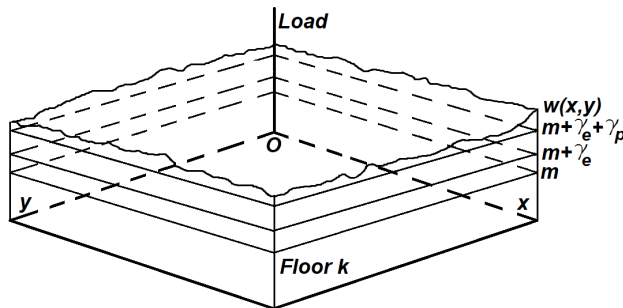


Figure 2. Components of live load model for a typical floor.

Equation (2) defines the variance of $w(x,y)$ which is computed as the sum of the variances of the involved random variables.

$$\text{var}(w(x,y)) = \sigma_e^2 + \sigma_p^2 + \sigma_{sp}^2 \tag{2}$$

In Equation (2) σ_e^2 is the variance of γ_e , σ_p^2 is the variance of γ_p , and σ_{sp}^2 is the variance of $\epsilon(x,y)$. To fit the model, it is necessary to use field measurements which give information about the magnitude, position, and duration of live loads. To do so, at a given instant of time, each measured unit load $U(A)$ linked to a given position is associated to the load of the model. Here, A is the variable describing the area of the floor. Main parameters of the load process can be obtained using adequate relationships based on $w(x,y)$ [3,5].

Equation (3) allows to compute the value of the unit load $U(A)$ occurring upon a given floor area dimensioned as $a \times b$. Equation (4) allows to calculate the expected value of the unit load $U(A)$ using the expected value of the loads given by the model $E(w(x,y))$.

$$U(A) = \frac{1}{A} \int_0^a \int_0^b w(x,y) dx dy \tag{3}$$

$$E(U(A)) = \frac{\int_0^a \int_0^b E(w(x,y)) dx dy}{A} = \frac{m \int_0^a \int_0^b dx dy}{A} = m \tag{4}$$

It has been observed that the larger the floor area the lower the live load mean value; also, due to the moving nature of instantaneous and extraordinary load in partial areas (*i.e.* aisles) and the quasi-static nature of sustained load in the rest of floor area, the covariance of $\epsilon(x,y)$ will fit well to an exponential decay form and the variance of the unit load $U(A)$ can be computed using the Equation (5) [3,6].

$$\text{var}(U(A)) = \frac{1}{A^2} \int_0^a \int_0^a \int_0^b \int_0^b \left[\sigma_e^2 + \sigma_p^2 + r_c * \exp\left(-\frac{r^2}{d}\right) \right] dx_0 dx_1 dy_0 dy_1, \tag{5}$$

where d is a decay constant, r is the horizontal distance between any two points and r_c is the correlation coefficient between the load values of points $A(x_0, y_0)$ and $B(x_1, y_1)$. In the case that points A and B are on the same floor, $r_c = 1$. When points are located upon different floors, r_c must be computed using the data obtained for n floors. The solution of Equation (5) can be obtained using numerical procedures and expressing the best fit of results in the form of Equation (6).

$$\text{var}(U(A_n)) = \sigma_e^2 + \frac{\sigma_p^2}{n} + \frac{\sigma_u^2 * K(A)}{n * A} + r_c * \frac{\sigma_{sp}^2 * K(A)}{n * A} * (n - 1). \tag{6}$$

Equation (6) facilitates the computation of the variance of the unit load using the variance using the variance of each part of the load process defined in Equation (2) and the variance of field data σ_u^2 . It is necessary to obtain the best adjustment for the parameter $K(A)$ and consider the number of floors n .

To include the effect of live load upon the internal forces of any structural element, a physical concept named “influence surface” is very helpful. The influence surface of a given internal force allows to study how the position and magnitude of a load applied upon a floor can influence the vector space characteristics. For example, when studying the bending moment generated upon the center of a given beam, the magnitude of $I(x, y)$ will indicate the relative value of that effect when the load is applied at any point $P(x, y)$. Then, the larger the $I(x, y)$ value, the larger the influence of the chosen position of the applied load. Values of $I(x, y)$ can be normalized making $x = x/L_x$ and $y = y/L_y$ being L_x and L_y limited by the mathematical domain associated to the studied floor area. If $F(A)$ is the total effect value, *i.e.* the bending moment expressed in KNm, main parameters of the studied effects can be computed as Equation (7), Equation (8), and Equation (9) [3,6].

$$E(F(A)) = L_y * L_x \int_0^1 \int_0^1 E(w(x, y)) * I(x, y) dx dy = m * A * m_1, \quad (7)$$

$$\text{var}(F(A)) = (\sigma_e^2 + \sigma_p^2) * A * m_1^2 + f * A \int_0^1 \int_0^1 I^2(x, y) dx dy, \quad (8)$$

$$m_1 = \frac{\int_0^a \int_0^b I(x, y) dx dy}{A}. \quad (9)$$

Equation (7) allows to compute the expected value of the studied effect ($E(F(A))$); Equation (8) allows to calculate the variance of the effect ($\text{var}(F(A))$); Equation (9) defines the modification factor of the effect m_1 . Once the maximum value of the studied effect has been determined, it is necessary to find what is the equivalent uniformly distributed load $Cu(A)$ which will produce the same value of effect. Solving numerically Equation (7) and Equation (8) and obtaining the best fit for new expressions, simplified equations can be derived, Equation (10) and Equation (11) [3,6,7,9].

$$E(Cu(A)) = m, \quad (10)$$

$$\text{var}(Cu(A)) = e + Ke * \frac{f}{A}. \quad (11)$$

Equation (10) states that expected value of uniformly distributed load $Cu(A)$ can be taken equal to the expected value of the load process $w(x, y)$; Equation (11) facilitate to compute the variance $\text{var}(Cu(A))$ using adequate parameters e , Ke , and f . Parameter Ke is related to the type of effect while parameters e and f are the same for any effect.

In this study, it is accepted that mean and variance of the sample to be good estimations of the population parameters whenever a low coefficient of variation is obtained. Using cumulative relative frequencies for loads and effects for each area, the goodness of fit to a Gamma distribution is verified using the Kolmogorov-Smirnov statistician for 1% and 5% significance [3,5,7]. During the service life of a dwelling, the floor areas can be subjected to changes of user (e.g. owners). Then it is necessary to investigate how the maximum loads behave as a function of the time. Here is assumed that the random loading process is homogeneous in time and space, *i.e.* it is accepted that successive sustained loads over time applied upon the same area are independent. Such sustained loads will have the same distribution of probability of the sustained loads at an arbitrary point in time; therefore, Gamma distribution is adopted to describe the variability of loads through the time [3,10].

Equation (12) indicates how to determine the cumulative distribution of maximum loads $FDA_M(\alpha)$, where α is the studied value of the load, M is the maximum sustained load value which hitch occurs in

the same floor over the lifetime of the dwelling T , α_s is the sustained load associated to a given percentile and $Cu(t)$ represents the equivalent sustained load uniformly distributed over time.

$$FDA_M(\alpha) = P(M \leq \alpha_s) = P[Cu(0) \leq \alpha_s] * P[Cu(t) \leq \alpha_s \text{ para } 0 \leq t \leq T]. \quad (12)$$

Using the conditional property rules for independent events, the first factor of Equation (12) is determined by integrating the Gamma function. To evaluate the second factor, it is accepted that changes of user follow a Poisson process complying with Equation (13) [3,10].

$$P[Cu(t) \leq \alpha_s \text{ para } 0 \leq t \leq T] = \exp(-v_\alpha \Delta t). \quad (13)$$

Equation (13) relates the exceedance probability of load with a Poisson process in which v_α is the average rate of events and $Cu(t) > \alpha_s$ and Δt is the interval of reference. The contribution of extraordinary loads can be simulated defining different combinations of furniture and people in specific areas [8,11]. It has been found that loads produced by agglomeration of people generate higher effect values, then the extraordinary loads must be estimated considering the possibility of such accumulation of weights depending of the available area [3,5,6]. Contribution of extraordinary portion to maximum loads can be determined with Equation (14).

$$FDA_{Me}(\alpha_e) = \exp[-v_e * t_e(1 - FDA_e(\alpha_e))]. \quad (14)$$

Equation (14) describes the distribution of the maximum extraordinary loads $FDA_{Me}(\alpha_e)$, where t_e is the average duration of a sustained load (time of occupancy of a user) that is related to the available time for the occurrence of various extraordinary loads for the same user, and v_e is the average rate of extraordinary events. The average rate of occurrence of extraordinary loads is $v_s = \frac{1}{t_e}$.

The highest total load can be computed as the sum of the maximum sustained load over the service life plus the highest extraordinary load occurring for the same user, or, as the sum of the highest extraordinary load over the lifetime plus the sustained load acting at the same instant. The occurrence of one mode excludes the other. The first combination described will give the most probable contribution for overload. Equation (15) allows to compute the maximum total load Ct as the sum of the maximum sustained load α_s and the maximum extraordinary load α_e *i.e.* [3,5,6].

$$Ct = \alpha_s + \alpha_e. \quad (15)$$

3. Results and discussion

Although housing solutions can be developed as buildings and dwellings, most of them are built as dwellings which usually have one or two stories (floors) and exceptionally three. Also, in Colombia, dwellings of low ($A \leq 70 \text{ m}^2$) and medium built total area ($70 \leq A \leq 140 \text{ m}^2$) are the most common. In this study, the population was chosen as the total amount dwellings with total built area lower than 140 m^2 existing in San José de Cúcuta, Colombia. Sampling characteristics were defined intentionally according to the consent of the owners.

There were chosen 10 dwellings classified as low built total area and 10 dwellings classified as medium built total area resulting the sample size equal to 20 dwellings. Sample was composed of 30%, 55%, 15% of dwellings of one, two and three stories high respectively. Due to the preponderance of low height constructions, only effects related to bending moment in slabs were considered. Geometrical information and current load values were taken measuring data from each dwelling of the sample.

From Figure 3, it is evident that load value decreases while the occupied area increases; also, a strong reduction in the dispersion of data is observed. These conditions lead to conclude that the relation between the variance of loads and the occupied area is well described by Equation (4), Equation (6), and Equation (11).

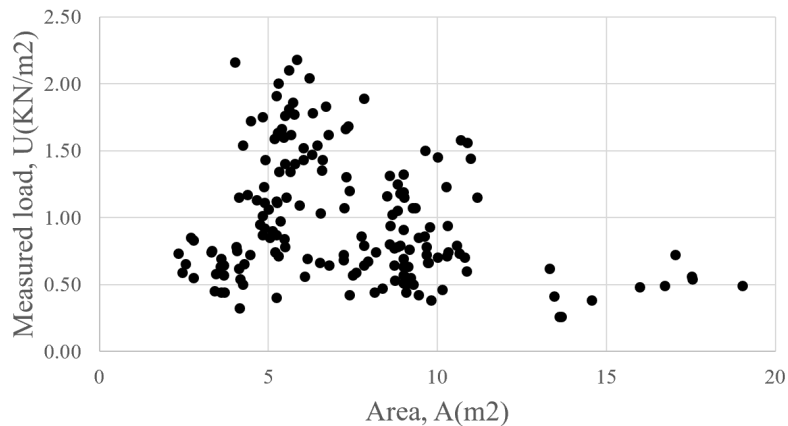


Figure 3. Field data of sustained load $U(A)$ obtained for 169 values of area of floor A . A decreasing trend of load with respect to an increasing occupied area is observed.

To capture information, formats and surveys were designed, available plans were reviewed, real geometrical measurements were taken, interviews with people and typical weights of furniture and people were studied. The field data are associated with concentrated sustained loads P_s , areas of cell A_c and times of changes of user. Maxima sustained load values were taken as percentiles of 0.90 and 0.99 while extraordinary loads values α_e were computed by making $FDA_{M_e}(\alpha_e) = 0.50$.

In this work, two effects were studied: Bending moment at the center of the span and bending moment at one end of the span. Effects related to axial load in columns or walls and shear at one end of the span were analyzed and found to be lower than bending moment effects. For this reason, such results are not shown. Obtained values of the parameter k_e for bending moment at the center and at one end of the span were 2.71 and 2.14 respectively. When evaluating user changes, it was found that the average time between changes is 11 years, resulting in a value of $v_s = 0.09$ users per year. In the case of extraordinary loads, information was not available then it was necessary to adopt a heuristic model.

Field data and Equation (1) to Equation (15) were processed using MATLAB [9]. The mean value of sustained loads resulted to be $m = 0.98$ KN/m². Also, it was found that the correlation decays to $\exp(-1)$ in a circle of 0.71m diameter which ratifies the hypothesis of spatial independence. Maximum loads obtained for the central bending moment effect are described by Equation (16) and Equation (17).

$$P_{0.90} Ct(A) = 1.14 + \frac{9.50}{\sqrt{A}}, \quad (16)$$

$$P_{0.99} Ct(A) = 1,56 + \frac{12.27}{\sqrt{A}}. \quad (17)$$

Equation (18) and Equation (19) allow to compute the maximum loads $Ct(A)$ obtained for percentiles 0.90 and 0.99 respectively when the effect studied is the central bending moment. Maximum loads obtained for the bending moment at the end of the span are described by Equation (18) and Equation (19).

$$P_{0.90} Ct(A) = 1.28 + \frac{7.97}{\sqrt{A}}, \quad (18)$$

$$P_{0.99} Ct(A) = 1,67 + \frac{10.49}{\sqrt{A}}. \quad (19)$$

Equation (18) and Equation (19) allow to compute the maximum loads $Ct(A)$ obtained for percentiles 0.90 and 0.99 respectively when the effect studied is the bending moment induced at the end of the span.

Most of world design codes state that live load must be expressed as a mean value plus possible variations during the service life. Such variations are commonly considered multiplying the medium load value by a load factor FL greater than 1.0. We have found that, for small areas of floor, let us say $A \leq 20 \text{ m}^2$, average values of FL are 1.40 and 1.84 for percentiles 0.90 and 0.99 respectively. In the case of larger areas those values become in 1.29 and 1.70. The average value of the expected maximum loads is equal to 3.29 and 1.82 KN/m² for low and medium areas respectively. Modern design standards state that FL must be 1.60 when mean value of live load is 1.80 KN/m² [1,2,12]. According to this, current design standards could have a slight deficiency when specifying design loads of about 15%. Perhaps the cultural baggage and customs of people have more preponderance in the physical effects of using structures. This matter seems to be a good topic to investigate in future works.

4. Conclusions

The use of Gamma and Poisson distributions are highly recommended to study the relation between the internal forces generated in structural elements and the temporal and spatial variability of live load. Practical expressions using simplified statistical parameters facilitate the analysis of a physical-mathematical model that is useful to forecast the live load trend in the future. From the analysis of field data, it is evident that the live load varies importantly in function of the size of the occupied area. In general, the larger the used area, the lower the live load.

In load distribution models, the correlation between load values on the same floor can be considered negligible. Live load levels required in current design standards are about of 85% the design loads forecasted in this study. It seems that customs of people can produce important variations in the physical effects which demand the structures.

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