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Teaching the concept of the uniform plane wave from mathematical modeling

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Teaching the concept of the uniform plane wave from mathematical modeling

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Abstract. The discovery of electromagnetic waves enabled us to understand electricity, magnetism, and optics under the same field; therefore, it is important for electronic and related engineers to understand the concept of the uniform plane wave, which represents the simplest case of an electromagnetic wave propagating through a medium. However, this concept is taught with a theoretical approach, shows a need for a methodological change towards experimentation to mitigate poor learning in students causing a high failure rate in electromagnetism courses. In this context, this work presents a methodological alternative for teaching the concept of the uniform plane wave, combining the CUVIMA model with the didactics of mathematical modelling, to link experimentation and provide cognitive support to the conceptualization of the physical phenomenon. For it, a single-group design is implemented, with pre-test and post-test, to know about previous ideas and identify the conceptual change achieved for the students. When applying the Student's t-test at a significance level of 5%, a significant difference is evident between the means of the study groups. It is concluded that there is a favorable and representative conceptual change in the students, with respect to the understanding of the physical phenomenon represented by the concept of the uniform plane wave.

1. Introduction

Uniform plane wave represents the simplest case of an electromagnetic wave propagating through a medium. Mathematically it is a particular solution of Maxwell's equations considering that the electric and magnetic fields are perpendiculars to the propagation direction, are in infinite planes, and have the same direction, magnitude, and phase. Although the uniform plane wave does not exist in practice, it is considered very useful in the study of the propagation of electromagnetic waves from both the theoretical and practical point of view [1,2], and therefore it is relevant for the compression of the physical phenomena studied in the electrical communications and antenna theory.

The teaching of the uniform plane-wave contemplates different challenges that the traditional transmissionist-behaviorist paradigm is unable to solve. Factors such as deficient and inadequate mathematical teaching make the modeling of electromagnetic phenomena complex and difficult and do not allow to achieve an adequate understanding [3-6], even students do not achieve the expected conceptualization even after successfully passing the courses of their careers and are unable to solve a set of tasks, exercises, and problems, and problems of the discipline [7].

For the above reasons, we propose to innovate teaching practice using the CUVIMA model and the didactics of mathematical modeling as a solution. The CUVIMA model organizes didactic activities involving all the actors in the teaching process and including the use of technological tools to perform experiments with the physical phenomenon in the laboratory or at home. This model admits the inclusion of the didactics of modeling, which is considered a highly effective tool in mathematics education research. The main contributions are the development of research skills and the synergy between the physical phenomena of a reality that is palpable, observable, and experimental with the related mathematics [8,9].

2. Methodology

The research develops three moments: first, to identify previous ideas; second, to construct the model of the uniform plane wave from experimentation; and finally, to identify conceptual change and significant learning; this within a pre-experimental design of a single group with pretest and post-test whose purpose is to analyze the implications and influence of the didactics of mathematical modeling in the understanding of the concept of the uniform plane wave and the physical phenomenon that it represents [10].

2.1. Identifying previous ideas

This work applies a pre-test before implementing the pedagogical strategy to find out the students' previous ideas. The pre-test is a questionnaire that uses the cognitive model "question-answer" to investigate students' conceptions of the concept of interest [11].

2.2. Construction the model of the uniform plane wave from experimentation

This work uses the CUVIMA model as a guide to design the didactic activities. The CUVIMA model suggests three stages: first identifying the concept to be developed, then selecting the technological tool to carry out the experimentation, and finally, designing the didactic experience from the mathematical modeling [12]. This work identifies the uniform plane wave as the concept of interest, selects a radiofrequency tone generator, a signal generator, a spectrum analyzer, and an oscilloscope to observe waves in the frequency and time domain as technological tools for experimentation and designs two sequences: the first one introduces the parameters of frequency, period, wavelength, amplitude and wave shape describing their behavior, graphical and mathematical representation in the time and frequency domain; the second one introduces the parameters of attenuation and phase constant to describe their implication in the propagation of electromagnetic waves. Figure 1 details the workflow of each didactic sequence.

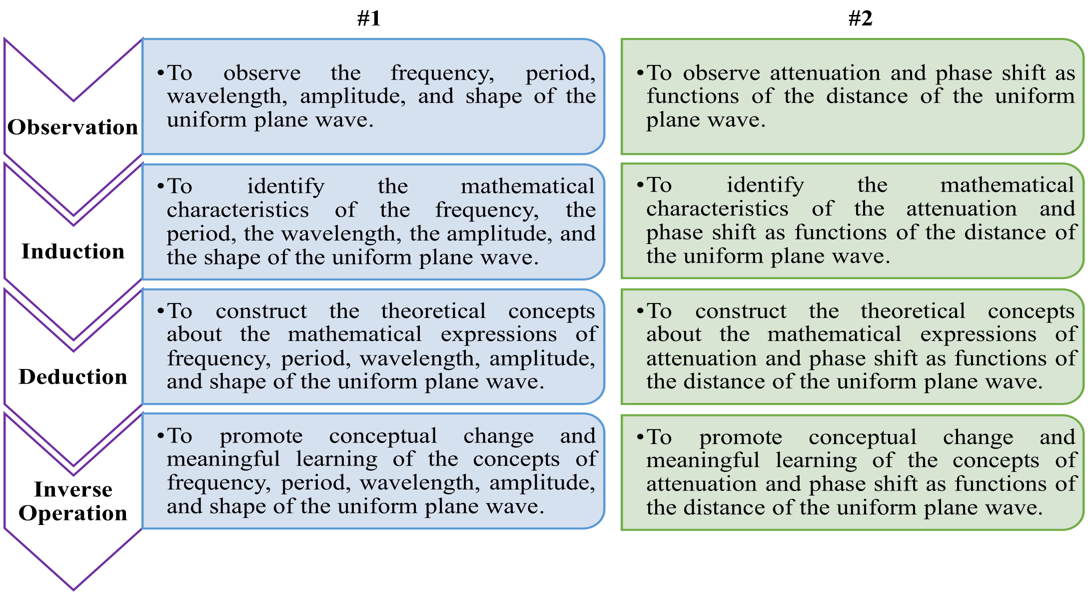

Figure 1. Workflow of didactic sequences.

If you do not have specialized technological tools, an alternative is to use two Bluetooth technology devices by installing an online tone generator and a mobile application on each one; in this way, it is possible to visualize the spectrum and take samples by reading the signal reception level. It is important to remember that the wave properties are equivalent regardless of the frequency spectrum.

The workflow of the didactic sequences consists of four stages: observation, inductive interpretation, deductive interpretation, and inverse operation exercises. Each stage is described below.

2.2.1. Observation. the student generates waves to identify properties such as frequency, wavelength, amplitude, and wave shape. It is convenient to mention that a single uniform plane wave does not accurately represent the propagation of electromagnetic waves; however, an electromagnetic wave is the sum of uniform plane waves from Fourier analysis [13]. Therefore, an analysis of the propagation of electromagnetic waves using the concept of the uniform plane wave lays the foundation for the analysis of any electromagnetic signal used in telecommunications.

2.2.2. Inductive interpretation. The student constructs mathematical expressions to establish a symbolic and abstract relationship with the observed phenomenon. This stage uses critical pedagogy to provide feedback and reinforce experimentation with a theoretical conceptualization of the uniform plane wave. Equation (1) and Equation (2) are the expressions to be constructed by student; where E and H represent the magnitude of the electric and magnetic fields respectively, E_0 and H_0 are the maximum amplitude of the electric and magnetic field waves, $e^{-\alpha k}$ is the attenuation factor, α is the attenuation constant, k represents the special dependence, sine function (sin) and cosine function (cos) indicate that the wave has a harmonic behavior when propagating in space, ω is the characteristic frequency, t shows the time dependence, β is the phase constant, φ represents the initial phase shift of the electromagnetic wave at any observation point and, finally a_E and a_H reflect the vector directions of the electric and magnetic field respectively.

$$E = E_0 e^{-\alpha k} \sin(\omega t \pm \beta k \pm \varphi) a_E \text{ ó } E = E_0 e^{-\alpha k} \cos(\omega t \pm \beta k \pm \varphi) a_E, \quad (1)$$

$$H = H_0 e^{-\alpha k} \sin(\omega t \pm \beta k \pm \varphi) a_H \text{ ó } H = H_0 e^{-\alpha k} \cos(\omega t \pm \beta k \pm \varphi) a_H. \quad (2)$$

2.2.3. Deductive interpretation. The student reflects on the mathematical model and performs theoretical experiments to observe the behavior of the uniform plane wave by varying parameters of distance, frequency, amplitude, attenuation, and phase. At the end of this phase, the student can associate the value of the mathematical parameters with the graphic representations of the electromagnetic wave.

2.2.4. Inverse operation exercises. In the experimentation, the student generates electromagnetic waves and identifies values of the mathematical model associated with the uniform plane wave. Therefore, in the inverse operation, the student proposing a graphical representation of the electromagnetic wave from theoretical values. This phase restructures previous ideas and promotes a conceptual shift towards more scientifically acceptable concepts [14].

2.3. Identifying conceptual change and meaningful learning
This work applies a post-test with the same structure as the pre-test. Generally, the pre-test and pos-test are the same questionnaires applied at different times during the research, allowing the analysis of changes in student learning and evaluating the proposed teaching methodology. The results are analyzed using the Student's t-test for correlated samples, comparing the statistical mean between the pre-test and the post-test [10].

3. Results
This work applies the proposed methodology to 130 students of the Electronics Engineering Program of Universidad Francisco de Paula Santander. The observation window is from the second semester of 2018 to the first semester of 2021. The results obtained are described below.

3.1. Preconceptions about the uniform plane wave concept

The pre-test consists of seven questions and identifies students' representations of the uniform plane wave concept. The answers are classified into two categories: inadequate definition and approach to the formal concept. Figure 2 shows the results obtained for each question.

Figure 2 shows that inadequate perceptions prevail over adequate perceptions. Analyzing the results according to the mean, 71.2% of the students have inadequate perception of the uniform plane wave, and 26.8% of the students achieve a formal approach on a few questions. Analyzing the answers qualitatively, the students define the electromagnetic wave in terms of its applications or effects that it suffers when propagating without defining the physical phenomenon it represents. Furthermore, some students think that electromagnetic waves are only transmitted through a material medium or exclusively through free space. On the other hand, some express that the plane wave is only a concept applicable to propagation in a vacuum. Finally, some of them do not respond or write nomenclature no mathematical or physical coherence.

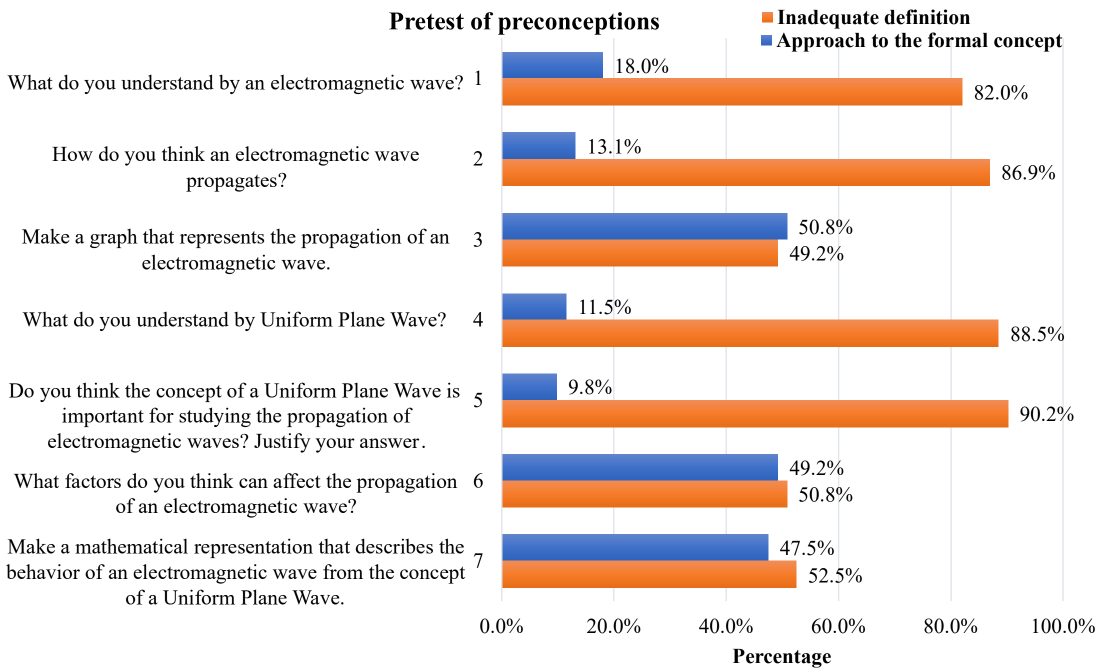

Figure 2. Results per question of pre-test.

3.2. Preconceptions about the uniform plane wave concept

The mathematical model of the uniform plane wave consists of four parts: the waveform, the frequency, the amplitude, the phase, and the attenuation. To guide the student in identifying the waveform, its relationship with frequency and amplitude, experiments are carried out using a tone generator and a spectrum analyzer, varying the mentioned parameters. The intuition is to use Fourier analysis to deduce that the unit impulse signals seen in the spectrum analyzer are sinusoidal functions in the time domain. The student also performs a similar experiment using an online tone generator in conjunction with a mobile application to view the spectrum; using an oscilloscope and a low-frequency function generator to reinforce the experience.

Figure 3 shows the equipment and technological tools used for experimentation. In this phase, the student uses Excel software to make graphical representations and infer concepts from the waveform observed in the experiments. First, the student identifies the linear relationship of the phase constant (β) with the distance and generates the corresponding mathematical expression. Then, the student identifies the negative exponential relationship between the attenuation constant (α) with the distance and its effect on the wave's amplitude, carrying out a day of measurements of the signal reception level between two devices for different distances generating the mathematical expression correspondent.

Finally, the student writes a mathematical expression with all the relationships found experimentally; the teacher expects the result to correspond with Equation (1) and Equation (2). Figure 4 shows the experimental results for the mathematical modeling of the uniform plane wave.

Figure 4(a) shows the shape of the uniform plane wave in a temporal representation, this experiment helps the student to infer variables such as period and frequency. Figure 4(b) shows the shape of the uniform plane wave in a spatial representation, this experiment helps the student to infer variables such as wavelength; these two figures demonstrate the spatial-temporal dependence of electromagnetic waves as they propagate. Figure 4(c) shows the relationship between phase change and space in the wave, this experiment helps the student to infer the linear relationship between phase constant and space in the mathematical model of the uniform plane wave. Finally, Figure 4(d) shows the relationship between the attenuation and the space of the wave, this experiment helps the student to infer the negative exponential relationship between the attenuation constant and the space in the mathematical model of the uniform plane wave.

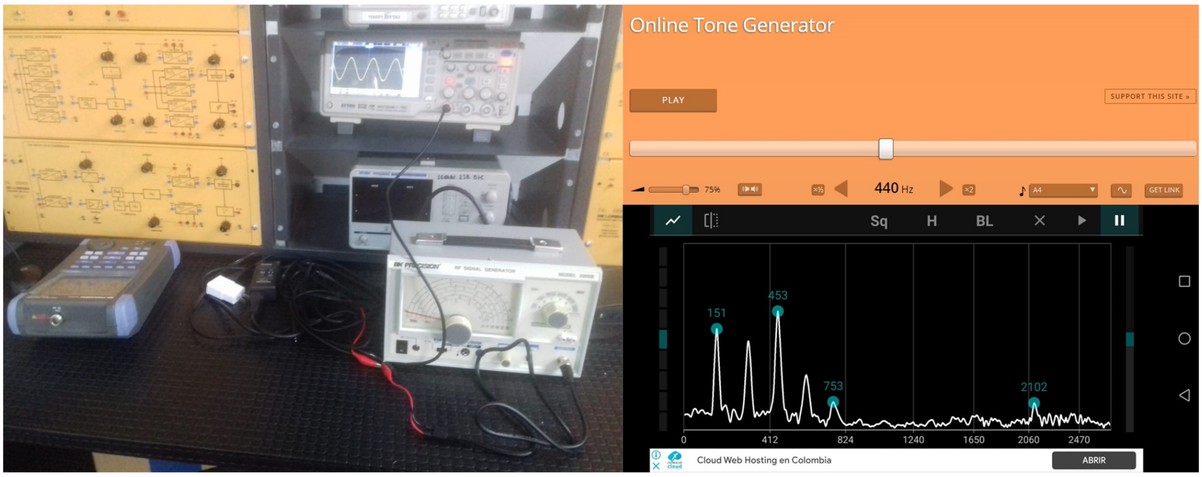

Figure 3. Equipment and tools for experimentation.

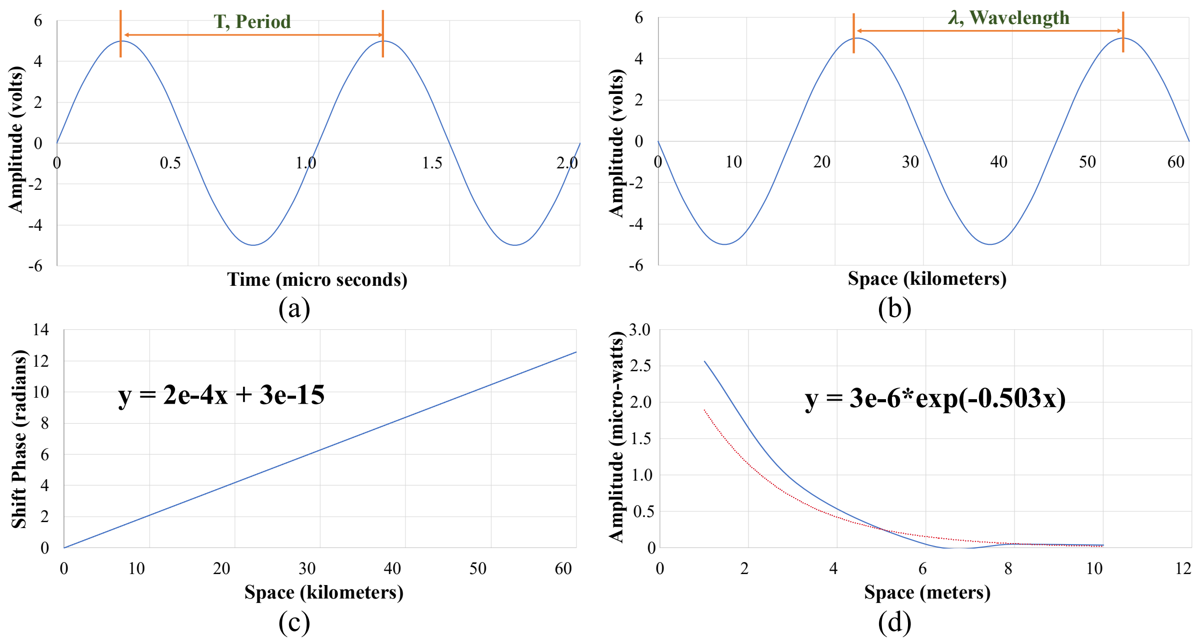

Figure 4. Results obtained in the didactic experience; (a) uniform plane wave in time; (b) uniform plane wave in space; (c) relationship between phase shift and space; (d) relationship between attenuation and space.

3.3. Conceptual change and significant learning achieve

The post-test has the same questions and the analysis of the answers as the pre-test. The post-test measures the conceptual change achieved by the students and evaluates the pedagogical proposal. Figure 5 shows the results obtained for each question. Finally, this work applied the Student's t-test to the pre-test and post-test obtained as a result, for a significance level of 5%, a p-value of 0.000051 for both categories, which indicates that the pre-test and the post-test scores are statistically different.

Figure 5 shows a considerable percentage decrease in all inappropriate responses and an increase inadequate response. Analyzing the results according to the mean, 13.8% of the students have inadequate perception of the uniform plane wave, and 86.2% of the students achieve a formal approach on a few questions.

These results are evidence of a conceptual change in the modification of representations of the uniform plane wave. Prior to the didactic activities, the students presented answers erroneous and, in some cases, did not respond, however, in this phase of the research, the representations were modified, presenting the plane wave as a harmonic wave dependent on time and space with attenuation and phase effects depending on the characteristics of the medium, of frequency and distance.

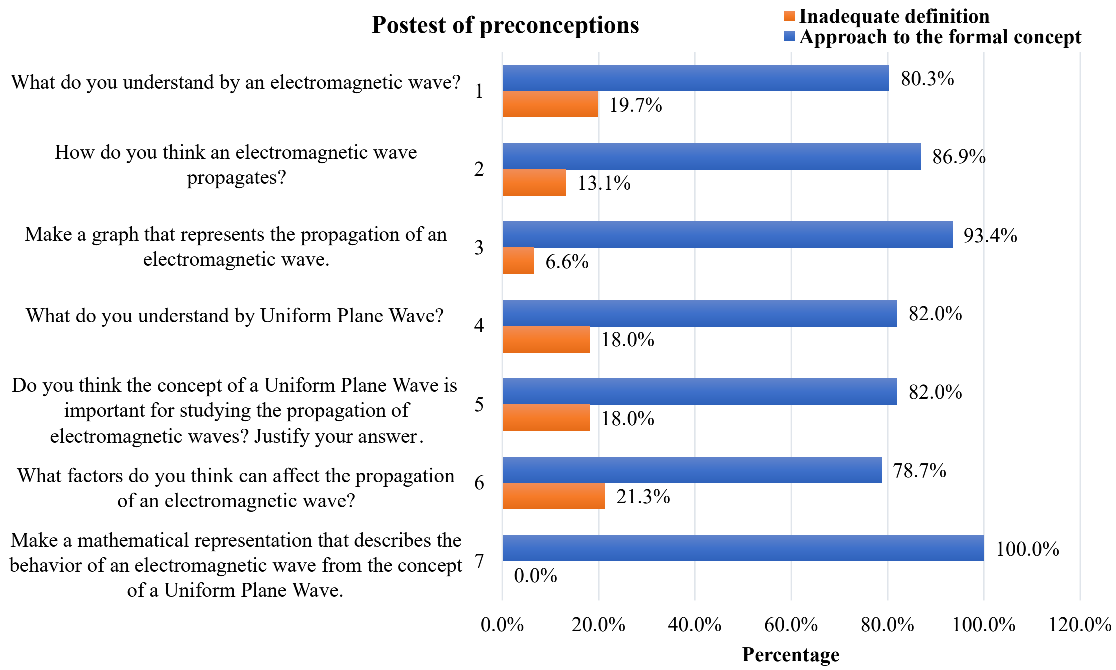

Figure 5. Results per question of post-test.

4. Conclusions

This work analyses the previous ideas of the students of the electronic engineering program at the Universidad Francisco de Paula Santander, Colombia, and development a methodology based on the CUVIMA model, introducing the didactics of mathematical modeling as a strategy to mitigate the shortcomings in the teaching of the concept of the uniform plane wave and obtaining satisfactory results.

The didactics of mathematical modeling in pedagogical practice manages to modify the conceptual ideas of the uniform plane wave through the application of didactic sequences that involve experimentation as the principal agent of change. Therefore, the previous concepts of the students concerning the propagation of electromagnetic waves changed through the progressive development of complex conceptual structures, where the students achieving to describe and to define the behavior of waves from the concept of the uniform plane wave, argued from the formal theory of the phenomenon and leaving aside the primary ideas conceived through the theoretical study of the mathematical concept.

Some students do not present satisfactory progress. However, this work shows the evolution in the construction of mathematical and physical concepts through the inductive and deductive interpretation applied through the didactics of mathematical modeling and supported using digital technology, resulting in the motivation and generation of better learning.

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