Original Article

Mathematical Modelling As A Tool To Understand Everyday Situations

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Abstract

This research reports the effects derived from the application of mathematical modelling in classroom work within the development of mathematics classes, which allowed the resignification of the concept of linear function, making use of various semiotic registers of representation applied to a wide range of everyday situations. The fieldwork was based on the application of a didactic sequence to two groups of ninth grade students from a public educational institution located in the capital city of northeastern Colombia. The students were expected to build a mathematical model that would fit the data provided framed in the situations in context in order to contribute to the re-signification of mathematical knowledge. The results show that the practice of modelling favours the motivation of students in relation to mathematical knowledge, giving it meaning and significance when confronted with real situations.

Keywords: Mathematical modelling, linear function, semiotic representations, resignification of knowledge, linear function.

Introduction

Modelling and its applications, teaching and learning at various levels of education has gained importance in recent years in view of the worldwide growth in the importance of the use of science, mathematics and technology in everyday life (Lesh, et al., 2010). A change is needed in mathematics education focused on the development of the competences needed to use mathematics to solve real-world problems as defined in the Curricular Guidelines in Mathematics (Gómez, 2010), i.e., what is taught in the classroom needs to be useful and practical and not seen as knowledge far removed from reality; on the contrary, its usefulness in proposing alternative solutions to real-world and everyday problems in the context of the students must be demonstrated (Córdoba, 2014). In this sense, Biembengut & Hein (2004) state that mathematical modelling as a method of teaching mathematics at all levels of schooling allows students not only to learn mathematics in a way that is applied to other areas of knowledge, but also to improve their ability to read, interpret, formulate and solve problem situations, as highlighted in the work of Prada et al. (2020) where a binary logistic regression model is proposed to determine the physiological effects of Covid-19 in members of an academic community.

Blum & Borromeo (2009) recognise modelling as an essential tool for helping students to better understand the contexts in which they operate and to promote the development of certain competences and appropriate attitudes towards mathematics. Along these lines, Blum et al. (2007) affirm that modelling plays a role that gives importance to the development of competences in students within the process of constructing models, their interpretation, argumentation and validation with the respective real situations, as they sustain it. For Bassanezi (1994), the use of modelling in teaching leads to the learning of mathematical contents that are connected to other forms of knowledge in order to develop a particular way of thinking and acting, producing knowledge through abstractions and formalisations, interconnected to empirical phenomena and processes considered as problematic situations.

Likewise, the modelling process, as part of school education even from an early age, allows students to acquire competences with the aim of analysing, investigating, sustaining and establishing mathematical models outside the classroom; to establish solid cognitive roots for the student's conception of some basic mathematical concepts, the recognition and value of students' experiences in the construction of mathematical concepts, to describe, analyse and extend the understanding of some phenomenon or situation of daily life in order to motivate the work with mathematics (Blomhøj, 2004).

According to the above, modelling can serve other purposes, as it provides cognitive support to students' conceptualisations, places mathematics in the culture and forms a critical attitude towards pre-established models. In Arrieta et al. (2007), in which a modelling practice was carried out (on the pollution of a river), some factors are established in which the importance of modelling practice and its usefulness in the development of scientific competences are highlighted. Finally, D'Ambrosio (2009), confers importance to modelling as a strategy par excellence of human beings for the creation of knowledge, as it allows validating and making predictions about the behaviour of the system being modelled and the possibility of controlling it.

Modelling in the mathematics Classroom

One of the characteristics of mathematics courses at different educational levels is the scarce or insufficient link with modelling activities that manage to articulate mathematical contents with real situations or phenomena close to the daily life or experiences of students in such a way that mathematical knowledge is placed on a different plane to the theoretical and conceptual, so that it emerges as an important tool and support in other areas of knowledge. It is common for modelling to be associated as a final activity. This is evident in the texts when application or modelling problems are presented at the end of the chapter. Thus, it is assumed that these application problems can only be carried out when the mechanisation of processes has been previously acquired, i.e. it is considered an application of the mechanised (López et al., 2015), but not a source that allows the resignification of school mathematical knowledge or that can be carried out at the same time as the development of the courses.

Although mathematical modelling is considered in the curricular designs of mathematics courses in general (Sarmiento-Rivera et al., 2020), the direct linking of teaching and learning with modelling practices with data from real situations that promote the articulation of mathematics with other fields of knowledge and at the same time allow giving meaning and life to mathematical knowledge is still insufficient in the educational context (Villa-Ochoa, 2015). The importance of experimental activity in modelling practices in mathematics teaching has already been considered by some authors (Rodríguez & Quiroz, 2016; Castrillón-Yepes et al., 2020; Villarreal & Mina, 2020).

Despite the importance that has been given to modelling activities, in the mathematics classroom there is still no link and integration of these in the mathematics curricula, since, in school contexts, these processes are not very common (Córdoba & Ardila, 2015). In this same sense, for Arrieta & Trinidad (2009), the school has minimised mathematical creation based on experimentation in the laboratory and is dismissed in the classroom, so that modelling can be seen through the linking of the school's practices with its environment. The practice of modelling in this sense should be based, as far as possible and according to the resources available, on a real activity close to the students that allows them to become actively involved in the experience and experimentation of mathematical activity.

Re-signification through modelling

Resignifying mathematical knowledge is not about giving it a new meaning but about transforming knowledge into functional knowledge in and with the human activities it transforms, which may be regulated by institutional and cultural aspects in a particular context manifested in the use of knowledge within a specific situation (García, 2018).

In this way, modelling is a diversity of daily, professional or scientific practices (Arrieta & Díaz, 2015) when exercised by human groups (students and teachers) in a particular context and under certain circumstances, it promotes interactions, negotiations and consensus that give sense and meaning to school mathematical knowledge, which is constructed and reconstructed in the situations of interaction that occur in the classroom, producing resignifications of meanings within a process of negotiation (Pezoa & Morales, 2016).

This is how resignifications are produced through modelling to the extent that, in the exercise of this practice, there is a specific, situated and intentional use of the linear function, within the context of two everyday situations, which makes it possible to give added value to this mathematical concept and its use, no longer isolated and decontextualised from reality but linked to it in an interactive process in which the students, through their own experience, seek the construction of a mathematical model in which the variables are related, providing new uses and meanings to school mathematical knowledge, thus enriching meanings at the same time.

This resignification is achieved from the confrontation between what students already know or think they know, between what they intuit or what their common sense suggests to them and the results obtained from a real situation in which they have been the main actors and have had the possibility of interacting with their peers and with the teacher. It is in this sense that resignification gives value, interest, new questions and new visions to mathematical knowledge, modelling the concept of change or variation, enriching its meaning when a group of students in a particular context exercises the practice of modelling.

Interactions in mathematics class

Classroom interaction corresponds to the representations and ways in which different elements that make up the teaching and learning process relate and communicate with each other (teacher, student and knowledge) in addition to the socio-educational environment in which they participate (Martínez-Maldonado et al., 2019). This means that knowledge, in this case school mathematics, as a collective construction, occurs in the interaction from the teacher to the student, generating modifications in the cognitive structure, provided that these interactions conform to certain criteria, mentioning among them intentionality and reciprocity, meaning and transcendence (Villalta-Paucar et al., 2018). Martínez et al. (2016), state that the teacher, the student, the object of knowledge and the teaching objectives are the elements of any educational practice, but it is the interaction between them that determines that practice and is at the same time the intrinsic element of the effectiveness of any educational environment, especially the mathematics classroom.

In social interactions, the daily experience of the social world is revealed, distinguished and perceived in a given scenario, which in this case corresponds to the mathematics classroom, precisely because it is in these relationships where behaviours are produced that are related to the demands, conflicts and influences of society and culture (Chajin-Mendoza, 2012). Therefore, those that contribute to the construction and re-signification of school mathematical knowledge in the context of the mathematics class are of interest, such as the processes of student and teacher association, between which there is an exchange, an orientation and an affectation of behaviour. These processes of interaction between the members of a group generate a network of relationships that constitute a learning space, in which, and through which, the co-organised subjects support each other (Gómez-Valderrama et al., 2020) and a space for facilitating exchange is provided (Radford, 2011).

In classroom interactions, discourse plays a central role constituted by communicative practices that generate the production and transaction of intentions and meanings in socially and culturally situated interactions (Planas, 2011). Therefore, it makes sense to analyse interactions through the discourse assumed in its various expressions or manifestations, whether verbal or written, each of which in turn allows us to analyse and account for the nature of the arguments and knowledge used not only in the construction of school mathematical knowledge (Molina, 2016), but also in its resignification. In this experience, the aim is for interactions to be promoted in the exercise of a modelling practice and more specifically in the exercise of the modelling practice of the phenomenon of cooling. In this way, oral and written productions are considered as forms of expression of these interactions and therefore susceptible of being analysed.

Methodology

Design

The method chosen is from a qualitative perspective as it implies a thorough observation in the natural context where these interactions and behaviours take place and enables a permanent communication with the subjects to be studied, directly or indirectly, in order to know their perceptions and conceptions about the phenomenon or event and to achieve the most detailed and complete descriptions possible of the situation, in this case, the resignification of school mathematical knowledge in the exercise of modelling, which is oriented towards the ethnographic method.

The aim is to record and analyse in as much detail as possible the interactions that take place (within teams, between teams and the whole group) and how these favour the re-signification of mathematical knowledge, in this case the linear function that is applied in a variety of everyday situations, and not simply to describe these interactions and events through an argumentative and discursive scenario (in the mathematics class) in which students and teacher, as a human group, interactively construct arguments, establish conjectures, explanations and create tools (the models) and meanings from interaction with a real world phenomenon, as a human group, interactively construct arguments, establish conjectures, explanations and create tools (the models) and meanings from the interaction with a real-world phenomenon.

Materials and participants

In carrying out the pedagogical activity, the setting for its development was a conventional classroom of a public institution. The population is the totality of ninth grade students of the educational institution (25 students) and with them four working groups were formed with six students, groups A, B and C, while group

D had seven students. The modelling activity was carried out with all the students, who were aged between 13 and 16 years old, with an average of 14.7 years and a standard deviation of 0.96 years. All of them live in stratum two dwellings, where 65% of the households are functional, in contrast to the others whose composition is very diverse. The economic activities carried out by the people who work in their households are mainly concentrated in independent commercial activities, construction helpers or domestic employees.

Pedagogical activity

A pedagogical intervention activity was carried out in which four everyday situations were included that could be modelled by means of the linear function and in which its approach is in everyday language. The pedagogical intervention lasted two hours.

Stages of pedagogical activity

The pedagogical intervention revolves around the application of a didactic sequence in which four everyday situations are proposed that can be modelled by means of the linear function with the particularity that in all cases the statements were given in everyday language:

Situation 1. The number of calories a person burns on a bicycle is proportional to the speed that he/she acquires assuming the same level of intensity. If a person exercises at a speed of 2.5 metres per second he/she burns 150 calories in one hour, but if he/she increases the speed to 6.5 metres per second he/she would burn 550 calories in one hour. Determine the function that represents the number of calories burned in an hour as a function of speed. From this function, determine the number of calories that would be burned in one hour of exercise if the speed were 4.5 metres per second.

Situation 2. Don Carlos owns the biggest shop in the neighbourhood where he sells all kinds of products. If the price of a kilo of cheese is $15000 he sells 30 kilos a week, but during the rainy season, the roads are affected, which means that he has to increase the price of the kilo of cheese by $3500, resulting in a reduction in weekly sales of 10 kilos. Assuming that the quantity of cheese sold weekly is inversely proportional to the selling price, determine the algebraic expression that relates these variables, and then find out how many kilos of cheese would be sold if the price were $16500.

Situation 3. Carlos' grandmother makes vanilla ice cream to sell. Once she prepares them and puts them in the aluminium mould, she puts them in the freezer of her fridge. She has found that they take five hours to freeze if the temperature is 14° Celsius, but if the temperature drops 8° they freeze in three hours. Assuming that the time it takes for the ice cream to freeze in the mould is inversely proportional to the temperature of the freezer in the fridge, write the function that associates these variables. Can you help Carlos' grandmother determine what temperature the freezer in the fridge should be if she wants her ice cream to freeze in no more than two hours?

Situation 4. John has a motorbike that he rides to work. To fill the tank of the motorbike to its full capacity he needs $33250 because it holds 12 litres of petrol. John bought a new motorbike that is much bigger than the one he had, but now he needs $55400 to fill the 20-litre tank. Determine the value of one litre of petrol, but define the value to be paid if 15 litres were to be put in the tank of the new motorbike.

The didactic sequence was divided into four moments, each of which had a defined intentionality, and which are articulated in a coherent whole in order to give a certain order and consistency to the modelling practice as such: a) Moment 1. Analysis of the proposed situation in order to identify its characteristic elements; c) Moment

3. Construction of the mathematical model and its representation in the Cartesian plane where the coherent articulation of semiotic registers of representation is made visible; d) Moment 4. Discussion of results and verification of conjectures.

Results

The following is a qualitative synthesis of what was observed in the work teams during the development of the didactic sequence, which is why it is broken down at each stage.

Moment 1. Motivation and background knowledge

The aim of this first part is to mobilise the students' prior knowledge so that they dare to formulate conjectures about each of the proposed situations, to establish links with other similar real-world situations and to try to associate the proposed situation with topics they have seen beforehand. This is a first exercise that promotes interaction between them, argumentation, consensus-building and communication, but also serves to awaken motivation and prepare the ground for the other activities. Within each working group there was a wide range of opinions associated with each situation, some of which were coherent and based on interpretations of direct and inverse proportionality; while others mentioned various aspects that were not considered in the statement and which gave rise to interpretations such as, for example, the influence that the brand of bicycle or refrigerator could have on the response, or if the cheese travelled by another means of transport, for example, that was not exclusive, such as a bus with passengers, which would have the effect of reducing the price, or that depending on the station where the petrol is bought, the price of petrol varies.

Moment 2. Identification of variables

Each working team was subdivided into groups of three people grouped together on a voluntary basis. Afterwards, the teacher gave a general explanation of the procedure to be followed. This phase is very important for the students to participate actively as they are faced with a real situation. At this point, a variety of aspects came into play, for example, teamwork, respect for the diversity of opinions, leadership, reasoning skills, the set of experiences and experiences that each person has, among many other aspects. The activity begins with each student reading the statement and then analysing and identifying what they consider to be the variables of the problem and trying to write down in their own words the relationship between them. Characteristic elements that stand out at this point in the class are that, regardless of the situation proposed, the students recognise them as everyday situations or claim to have experienced them or to have some knowledge of them, but it is very difficult for them to express the situation using mathematical terms, such as, for example, the identification of variables within the context, i.e. identifying within the situation those who assume the roles of independent or dependent variable and their relationship.

Moment 3: Building the mathematical model

At this point in the didactic sequence it is hoped to mathematise the proposed situation, that is, to try to construct a mathematical model that fits the relationship between the data in order to subsequently determine possible effects on the dependent variable based on the intentional manipulation of the independent variable. In this sense, it should be emphasised that what is important are the interactions that emerge as the discussion develops in the group work dynamic itself, and that allow various elements to emerge that provide evidence of the resignification of school mathematical knowledge. From the students' work, the articulation of the semiotic registers of everyday language into a tabular system was seen as a strength, i.e. they propose a table based on the values given in the statements and then represent it on the Cartesian plane by locating the ordered pairs of points connecting them by means of a straight line projected to infinity.

The students have difficulty in determining the slope of the line and, consequently, the construction of the mathematical model, due to various aspects such as: a) incorrect interpretation of the variables in each statement, which affects the whole process from now on; b) of the groups that correctly selected the variables, they tried to calculate the slope of the line but made arithmetical errors; finally, c) all the students were unaware of the equation of the slope of the line, so they were unable to advance to the construction of the equation. At the graphic level, it was evident as a conception that the students projected the line to infinity, which is an error when analysing everyday situations, since the mathematical model allows interpolation from what is proposed, but it would be an error to speculate outside the values analysed.

Moment 4: Discussion of results and verification of conjectures

Arguments, explanations, conjectures, consensus and tools (models) are key aspects of the interactions that emerge in the discussion within each team. At this point, there is a first construction or elaboration that the students have to do, i.e. they must try to arrive at a model that fits the data. This comparison of results allows for the identification of weaknesses and learning needs, correct or incorrect ideas and previous concepts, strengths and weaknesses in the development of the activity. In the same way, students will be able to realise the importance of taking mathematics to other areas of knowledge and how school mathematical knowledge can be re-signified from a real modelling situation. This is the time for group discussion in which consensus is established, some concepts are institutionalised and work experiences are shared. As a balance of the activity, it was evident that the students try to explain what happens in each situation, but it is difficult for them to move on to the construction of a mathematical model that represents the relationship between the variables. At the end, the teacher analyses and constructs the mathematical model associated with each situation, explaining the step by step to the group, showing understanding and recognition of the importance of this knowledge in everyday life.

Students' evaluation of the activity

In the students' responses to the evaluation of the activity, the vast majority agree that there is a better understanding, that there is ease in the understanding of knowledge and above all that there is life in that knowledge. This situation creates in the students a greater confidence that makes them see mathematical knowledge as something close to them and whose understanding is possible, as the environment created is favourable and conducive to the development of a really effective and dynamic teaching and learning process. It is important to highlight that the students feel more confident when the activity is carried out with the accompaniment of the teacher, which is evidence of low self-confidence in what they do themselves.

The qualitative re-signification of school mathematical knowledge

Resignification, the result of interactions within a human group, allows us to give a new value not only to the main knowledge put into play but also to other additional and complementary knowledge that is needed to solve each proposed problem. It is at that moment that school mathematical knowledge is justified and acquires meaning for the student, and it is in the intentional and situated use that this knowledge is re-signified to a greater or lesser extent, but always in a continuous process. Although the procedural aspect is important in the sense that the students were able to identify, for example, that they had weaknesses in carrying out arithmetic and algebraic processes, this aspect is not fundamental, as it can be easily corrected. What is important is that the students were able to realise that school mathematical knowledge has other scopes and that mathematical knowledge can be constructed through interaction.

This gives validity to this practice, modelling, as a promoter of varied and significant interactions that allow us to re-signify previous knowledge, strengthen current knowledge and prepare the ground for future knowledge in a dynamic of construction, argumentation and collective discussion within a human group.

Discussion and conclusions

Modelling is not simply putting into mathematical symbols an extra-mathematical phenomenon, it is also, at the same time, a process of discovery of learning weaknesses, of ways of relating to knowledge, to others and to the environment; it is, if you like, the functional side of school mathematics, but functional in the sense that it promotes other forms of interaction and of re-signification and collective construction of mathematical knowledge. According to Buendía (2004), interaction as a fundamental axis of social practices is the common "place" in which the actors construct identities, meanings, their own realities and at the same time reconstruct their own cognition.

When considered as a practice, modelling involves aspects that favour interaction, re-signification and the construction of mathematical knowledge that occurs when students exercise these practices collectively and in an environment of permanent discussion, confrontation and consensus building, and not simply when modelling is taught as another content or simply as a strategy to solve problems.

The students' answers allow us to conclude that mathematical knowledge can be understood in a different way to the traditional one, no longer as a set of mathematical objects out of context and lifeless, in which concepts are the fundamental thing. But as an articulated whole in which the situated and intentional exercise of the practice of modelling favours the process of re-signification of this knowledge, via the dynamics of the interactions that emerge from this situation. Modelling as a practice is the means that brings them closer to the reality of their reality in which interactions play a fundamental role in allowing the development of new and meaningful processes of resignification that give life and dynamism to school mathematical knowledge, no longer isolated and detached from reality but integrated and functional in it.

Beyond teaching modelling as a specific content of the course programmes, the aim was to study the phenomena that occur around modelling practices, the interactions that emerge from discussions in the face of a modelling problem, the previous and current knowledge that circulates in the classroom when this type of situation arises, the different paths or alternative solutions proposed by the students, their arguments and justifications, their exemplifications, their assumptions and certainties, their learning needs and the strategies they seek to meet these needs, the teacher's guidance, his discourse and his discourse, and the strategies they seek to use to meet these needs, their arguments and justifications, their exemplifications, their assumptions and certainties, their learning needs and the strategies they seek to meet these needs, the teacher's orientations, their discourse and management of the situation, the form of institutionalisation of the school mathematical discourse, everything that comes together in the modelling situation and the reasons that justify the actions of others. Modelling practices in themselves are the pretext for identifying and discovering other elements consubstantial to the resignification and construction of mathematical knowledge and its functional character, functional in the sense that it can be permanently integrated and resignified in life (outside school) in order to transform it (Cantoral & Farfán, 2003).

Thinking of the mathematics class as a scenario in which the resignification of mathematical knowledge is the main activity and that this is the result of interaction between students and with the teacher as mediator, is what makes the practice of modelling a highly significant and motivating alternative for students in their learning process. In this sense, modelling practice should, as far as possible and according to the resources available, be based on a real activity close to the students that allows them to become actively involved and to live and experience the mathematical activity.

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