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An application of physics: simply supported bridges made of post-tensioned concrete and structural steel beams

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Abstract. Bridges represent an important application of physics capable of solving real transportation problems. Knowledge of convenience of different mechanical solutions when analyzing and designing bridge is needed. For these reasons, this work is focused on the study of convenience of using two types of bridges. Simply supported short-medium span bridges (30 m to 45 m) are usually excessively long when choosing reinforced concrete solutions and usually short for other types of structures such as cable-stayed or cantilever bridges. The suitability of simply supported bridges leads to the need of studying their cost benefit ratios. This work studies the cost benefit ratio for post-tensioned concrete beams and structural steel girders in simply supported straight bridges. Eight models built of type I sections were used in both cases to analyze the bridges using a software based on the stiffness method. Span of each bridge was set to 30 m, 35 m, 40 m, and 45 m. The convenience of each type of bridge was done comparing the total and the cost per linear meter of each solution (post-tensioned and structural steel). Comparison was done using material consumption, labor, and construction processes costs only. Also, allowable vertical displacement given by current bridge design standards was verified.

1. Introduction

Bridges are an important application of physics for solving problems of road infrastructure works. Structural configuration of bridges depends on the natural or artificial obstacles to overcome and their characteristics such as lengths, shape, and height. There are numerous types of structural systems for bridges made of different materials such as reinforced concrete, post-tensioned concrete and structural steel are used [1]. To guarantee comparable safety and functionality of bridges methodologies for analysis and design of bridges are stated in worldwide standards. Some of the most important standards are American Association of State Highway and Transportation Officials (AASHTO), Load Resistant Factor Design (LRFD), bridge design specifications [2]; American Concrete Institute (ACI), ACI 318S-14 [3]; "Asociación Colombiana de Ingeniería Sísmica (AIS)", LRFD-CCP 14 [4] and NSR-10 [5].

Structural design of a bridge must consider the economic feasibility without prejudice to its functional purposes [6]. Post-tensioned concrete and structural steel are the most common materials for simply supported short-medium span bridges having lengths between 30 m and 45 m. When dealing with the construction of bridges, post-tensioned concrete or structural steel are widely used as an economic structural system, although this depends on the bridge length and the amount of supports [7,8].

Several studies have been carried out in relation to the study of costs of design and construction of bridges. Haas [9] investigated the cost effectiveness of steel girders compared to conventional reinforced concrete girders used in bridge construction in South Africa. Similarly, Delgado and Zuñiga [10] reported a cost comparison between bridges with steel girders, reinforced concrete and post-tensioned considering the variation of the free length of the span. On the other hand, Almeida and Armas [11] studied the economic behavior of two types of bridges, comparing the superstructure of 30 m span bridges supported only in two ends and considering post-tensioned concrete beams and gantry beams, designed according to some international bridge design standards. However, there has not been found information reported for Latin-American countries related to the scope of the present study.

The purpose of the present investigation is to compare the cost of two types of materials when used in the construction of simply supported short-medium span bridges using lengths of 30 m, 35 m, 40 m, and 45 m. To do so, bridges designed with structural steel beams and post-tensioned concrete beams are studied to determine which of the solutions has the best economic viability while maintaining a similar level of functionality.

2. Methodology

The research was carried out around the economic and functional study of medium-short vehicular bridges, with lengths between supports of 30 m, 35 m, 40 m, and 45 m. The bridge deck, for all cases, was designed using a reinforced concrete slab with a lane width of 8 m. To involve the most used materials used in practice, in this study post-tensioned concrete and structural steel were used. Safety and functionality were considered to define comparative costs. Safety computation was based on to beams strength while functionality was related to allowable vertical displacements.

2.1. Determination of materials quantities from optimized structural design

Post-tensioned concrete beams were conformed using I-shaped sections. As shown in Figure 1, four post-tensioned beams were outlined to support a typical 18 cm-thick reinforced concrete deck. The stiffness method was used to obtain results for comparison. Figure 1 shows an example of the typical configuration of the bridge section. Dimensions of each post-tensioned concrete beam vary according to the free span of the bridge. The larger the span the bigger the dimensions of the beam section. Medium flange width (B) and total height (H) of beam section of were specified according to each case. Values of B = 0.75 m, 0.80 m, 0.93 m, and 0.95 m were set for span beams of 30 m, 35 m, 40 m, and 45 m, respectively. Similarly, values of H = 1.80 m, 1.95 m, 2.15 m, and 2.35 m were set for span beams of 30 m, 35 m, 40 m, and 45 m respectively.

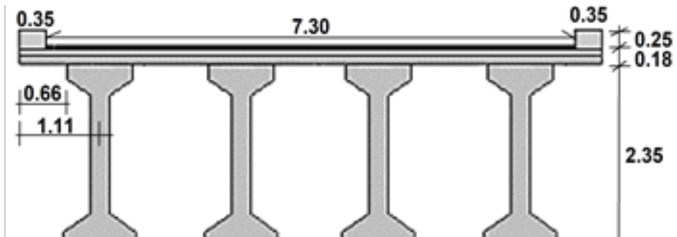

Figure 1. Example of vehicular bridge section for L = 45 m with reinforced concrete deck supported on post-tensioned concrete beams (*Dimensions in meters).

Each structural steel beam was designed using I-shaped section. Four steel beams were outlined to support a typical 18 cm-thick reinforced concrete deck. The stiffness method was used to obtain results for comparison. Figure 2 shows an example of the typical configuration of the bridge section. Dimensions of each steel beam vary according to the free span of the bridge.

The larger the span the bigger the dimensions of the beam section. Flange width (B) and total height (H) of beam section of were specified according to each case. Values of B = 0.40 m, 0.45 m, 0.48 m, and 0.53 m were set for span beams of 30 m, 35 m, 40 m, and 45 m respectively. Similarly, values of

H = 1.50 m, 1.65 m, 1.80 m, and 2.00 m were set for span beams of 30 m, 35 m, 40 m, and 45 m, respectively. Optimized structural design of each solution (post-tensioned concrete or structural steel beam) was carried out computing the minimum materials amount required for guaranteeing safety and functionality conditions stated in the Colombian standards [4].

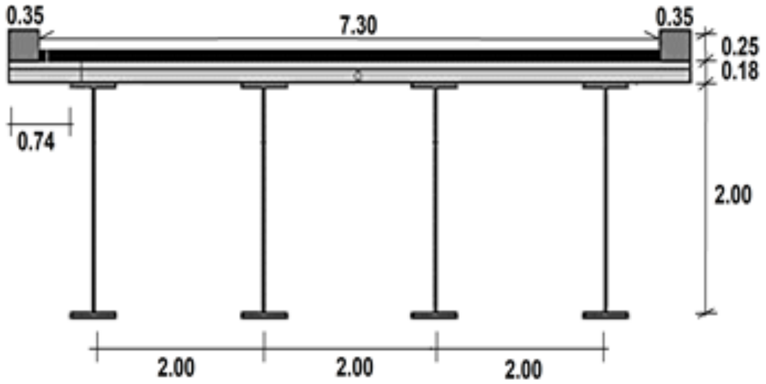

Figure 2. Example of vehicular bridge section for L = 45 m with reinforced concrete deck supported on structural steel beams. (*Dimensions in meters).

2.2. Determination of basic costs of materials

Unit price analysis was carried out for materials required from the results of the structural design of each solution. The costs were based on commercial prices, and they were computed using Colombian pesos (COP) of the year 2021. Costs of material, labor, and assembly of processed materials arisen from the construction of the studied bridges were considered when computing each unit price. Measurement units were based on the international system of units, being cubic meter (m^3), linear meter (m), and kilogram (Kg) the typical units.

2.3. Estimation of the trend line equation

To present results in a practical form, a descriptive equation which relates the trend of costs for each solution was estimated. Least squares method was used to get the best fit for costs associated to simply supported vehicular bridges designed using post-tensioned and structural steel beams. The least squares method calculates from the number of even numbers of experimental data (x,y), the slope m and intersect b values that best fit the data in one straight line. The expression is based on the equation of the line $y = mx \pm b$ (where m is the slope of the line and b is the cut-off point). Equation (1) and Equation (2) allow to compute the slope and the intersect point with vertical axis of the trend line, respectively [12].

$$m = \left(\frac{n*\sum(x*y)-\sum x*\sum y}{n*\sum x^2-|\sum x|^2}\right), \tag{1}$$

$$b = \left(\frac{\sum y*\sum x^2-\sum x*\sum(x*y)}{n*\sum x^2-|\sum x|^2}\right). \tag{2}$$

After calculating the factors m and b, the values are replaced in the expression $y = mx \pm b$. The expression shown in Equation (3) leads to compute the best adjust of a straight line which describes the trend of cost of vehicular bridges by increasing the length between supports from 30 m to 45 m [12]. This operation has been repeated for each type of bridges designed of post-tensioned concrete and structural steel.

$$y = \left(\frac{n*\sum(x*y)-\sum x*\sum y}{n*\sum x^2-|\sum x|^2}\right)x \pm \left(\frac{\sum y*\sum x^2-\sum x*\sum(x*y)}{n*\sum x^2-|\sum x|^2}\right). \tag{3}$$

3. Results and discussion

Table 1 summarizes the reference amounts used for comparison of physical efficiency of solutions made of post-tensioned concrete; bending moment demand has been computed considering the higher expected forces (loads) over the bridge following the current standards [2-4]. The ratio “demand/weight” has been calculated dividing the bending moment demand by the total weight of the bridge. Rows 3 to 8 of Table 1 refer to the material consumption in bridges made using post-tensioned concrete beams; demand/weight ratios vary from 225 KN×m/KN to 415 KN×m/KN following a parabolic trend in function of the span length which is consistent with the trend of dominant bending moment curve.

Physical effect of load lever and forces configuration seem to impose such second-degree trend; as for material costs, it is evident that concrete 35 MPa, prestressing steel and reinforcing steel are the most important cost makers. In contrast, concrete 28 MPa and safety railings are the lower cost contributors. A rising trend of materials consumption is observed; the larger the span the lower the cost variation between two successive spans.

Table 1. Reference amounts for bridges made using post-tensioned concrete beams.

Reference amount	Bridges 30 m	Bridges 35 m	Bridges 40 m	Bridges 45 m
Bending moment demand (KN×m)	44275	63571	89805	117549
Demand/weight (KN×m/KN)	225	280	341	415
Concrete 35 MPa for beams (m^3)	159	229	323	401
Concrete 28 MPa for braces (m^3)	6	7	8	9
Concrete 28 MPa for decks (m^3)	49	57	65	73
Reinforcing steel 420MPa (KN)	99	121	148	174
Prestressing steel for beams (Kg)	44	64	88	117
Safety railing (m)	60	70	80	90

Table 2 shows the reference amounts for bridges made with structural steel beams; in this case, the Demand/Weight ratios vary from 331 KN×m/KN to 612 KN×m/KN following a parabolic trend in function of the span length which is consistent with the trend of dominant bending moment curve. Again, as explained in previous paragraph. physical effect of load lever and forces configuration seem to impose such second-degree trend. Rows 3 to 8 of Table 1 refer to the material consumption in bridges made using structural steel beams. In this case, A572-01 G-50 steel, C 12×20.7 steel profiles and reinforcing steel are the most important cost makers are. A rising trend of materials consumption is observed; the larger the span the lower the cost variation between two successive spans.

When comparing results for demand/weight ratios from Table 1 and Table 2 it is concluded that, from the point of the physics (mechanics), the steel beams provide greater efficiency than post-tensioned beams. It is true because steel beams, which have a lower weight, can support a similar or larger demand than post-tensioned beams. For example, for 30 m span, one KN of structural weight supports a bending moment demand of 331 KN×m in the case of steel beams while post-tensioned beams only can support 225 KN×m. According to the previous reasonings, mechanical efficiency of steel beams is around 1.5 times that of the post-tensioned beams for all the studied spans.

Table 2. Reference amounts for bridges made using structural steel beams.

Reference amount	Bridges 30 m	Bridges 35 m	Bridges 40 m	Bridges 45 m
Bending moment demand (KN×m)	28898	37354	46895	57914
Demand/weight (KN×m/KN)	331	416	511	612
Concrete 28 MPa for beams (m^3)	49	56.53	64.60	72.68
Steel A572-01 G-50 (Kg)	655	851	1,053	1,317
Profiles C 12×20.7 for braces (Kg)	9	9	9	9
Steel profiles for shear connectors (Kg)	10	12	13	15
Reinforcing steel 420 MPa (Kg)	39	46	52	59
Safety railing (m)	60	70	80	90

Table 3 shows the basic materials costs computed for each material used in the design of the studied bridges given in Colombian pesos (COP) of the year 2021. First column of the table describes the material, second column refers the unit of measurement, and third column presents the cost per unit of reference for each material. The units of measurement are expressed using the International System of Units using cubic meters (m^3) for volume, linear meter (m) for length and (KN) for force. The higher unit cost corresponds to prestressing steel which is about three to six times more expensive than other types of steel. According to this, there must could be an important influence of the type of steel in the cost of each solution.

Table 3. Cost of construction materials for the year 2021.

Building material	Unit of measurement	Cost (thousands of COP)
Concrete 28 MPa	m^3	1056
Concrete 35 MPa	m^3	1132
Reinforcing steel 420 MPa	KN	1060
Prestressing steel	KN	6789
Steel A572-01 G-50	KN	2080
Steel profiles C 12×20.7	KN	1794
Steel profiles	KN	1794
Safety railing	m	748

Table 4 shows the resultant costs of each solution for the studied vehicular bridges. It is evident that bridges made using structural steel beams are up to two times more expensive than those made with post-tensioned concrete. However, it is important to clarify that this comparison is related only to superstructure cost of each bridge (cost of beams, deck, and braces). Further study is required in near future for other cost generators as the substructure costs and environmental costs. Also, as explained previously, steel beams guarantee a better use of the space and a lower weight which could influence importantly the efficiency of the required substructure of the bridge.

Table 4. Costs comparison for the different types of bridges (millions COP).

Building material	Post-tensioned concrete beams				Structural steel beams			
	30 m	35 m	40 m	45 m	30 m	35 m	40 m	45 m
Concrete 28 MPa	231.15	319.28	434.08	530.22	51.16	59.69	68.22	76.74
Concrete 35 MPa	179.99	259.59	365.86	453.48				
Reinforcing steel 420 MPa	105.06	128.18	156.62	184.15	41.73	48.68	55.64	62.59
Prestressing Steel	295.42	433.26	598.68	794.09				
Steel A572-01 G-50					1362.47	1768.91	2190.72	2738.39
Steel profiles C 12×20.7					15.39	15.39	15.39	15.39
Steel profiles					18.21	21.21	24.21	27.21
Safety railing	44.88	52.36	59.84	67.32	44.88	52.36	59.84	67.32
Total cost	682.60	940.19	1257.33	1584.91	1533.84	1966.24	2414.01	2987.65

Figure 3 shows the cost trend in function of the span length for each one of the studied type of materials. It is evident that the bridge superstructure cost depends upon the span length; such dependence relation indicates that the longer the span the larger the cost. Cost of superstructures made using structural steel beams is about two times the cost of superstructures made with post-tensioned beams Such difference keeps almost constant through the range of spans evaluated.

Table 5 presents the trend equation for the cost of vehicular bridge superstructure made using post-tensioned concrete and structural steel beams with lengths varying between 30 and 45 m. The information of the Table 5 was calculated using Equation (3). The slope (coefficient of X) shows that cost for solutions made using steel beams impose a more rapid cost increase than those made using post-tensioned beams ($96.184 > 60.482$). It is evident that there is good fit of obtained equations ($R^2 \cong 1$).

Figure 3, Table 4, and Table 5 have been built considering that the two types of vehicular bridges designed with post-tensioned concrete and structural steel beams must have a similar functionality; such functionality has been defined as a function of the physical movement, *i.e.*, a target displacement has been guaranteed for each condition. In general, a bridge made using post-tensioned concrete beams could cost around the half of the cost of a bridge made using structural steel beams. In summary, That, the construction of post-tensioned concrete vehicular bridges could have greater economic viability than those designed with structural steel.

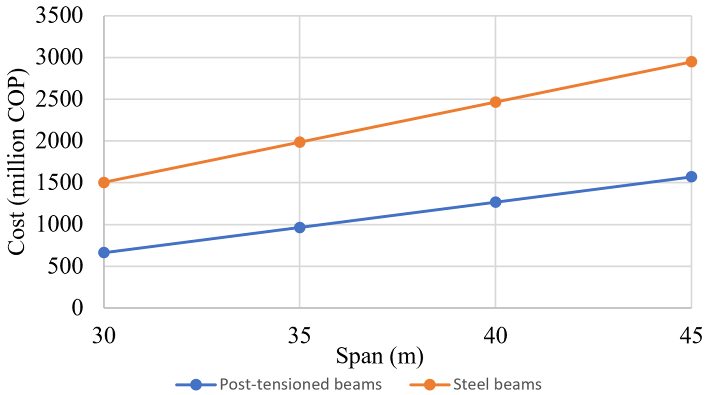

Figure 3. Cost comparison of vehicular bridge superstructure with spans ranging from 30 m to 45 m when designed using post-tensioned concrete or structural steel beams.

Table 5. Trend equations of the cost curves of short vehicular bridges ($30 \le X \le 45$).

Bridge girder material	Equation of the trend of the curve	R^2
Post-tensioned concrete	$Y = 60.482 * X - 1151.8$	0.9971
Structural steel	$Y = 96.184 * X - 1381.5$	0.9952

Y is the cost of the bridge superstructure in millions of Colombian pesos
X is the length of the bridge superstructure in meters ($30 \le X \le 45$)

4. Conclusions

The structural design of 4 bridges superstructures made using post-tensioned concrete beams and 4 bridges superstructures made using structural steel beams was carried out. Bridges superstructures designed with post-tensioned concrete girders resulted to be more economical than those designed with structural steel girders. In fact, solutions made of post-tensioned can cost around a half of what solutions made of steel beams cost. However, from the point of the physics (mechanics), the steel beams provide greater efficiency than post-tensioned beams because steel beams have a lower weight and can support a similar or larger demand than post-tensioned beams. Furthermore, mechanical efficiency of steel beams is around 1.5 times that of the post-tensioned beams for all the studied spans. Further analysis involving other important costs such as cost of substructure, social costs and environmental costs must be done before choosing any of the studied solutions.

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