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# Interpreting the slope of a straight line in kinematics graphs with school students

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**Abstract.** The interdisciplinary nature of knowledge is one of the objectives promoted in Colombia, fostering in students the ability to recognize that the curriculum does not work in isolation; if knowledge is complementary, its joint development leads to better academic performance. This research is oriented towards showing an application of the mathematical concept of the slope of the straight line in physics applications, favoring the understanding of the relationships between variables, as occurs with the graphs of kinematics. A group of eleventh grade students from a private educational institution filled in a questionnaire for data collection, based on three situations proposed to answer three open questions in which a reasonably justified answer was required. A part of the group correctly interpreted the graphs by describing the characteristics of the movement in each time interval, but only a few were able to associate the slope of the line as the velocity or acceleration as a function of the variables mentioned in the graph, although their argumentation was based on the angle of elevation of the line.

## 1. Introduction

Some authors as pointed out in [1] state that the origins of the theory of motion suggested by Galileo [2] was a bold and speculative theory about the phenomena he observed; while other researchers gave continuity to his ideas, arriving at the correspondence between the observed phenomenon and the theory, in subjects such as the motion of projectiles, free fall, pendular movements and the nature of the vacuum, as reviewed in the work of [3]. These early works gave rise to the branch of mechanical physics known as kinematics, which deals with the study of the characteristics and magnitudes of motion, such as the speed or distance travelled by a mobile in each time, or the acceleration experienced by a body as it moves over a surface, among others.

When dealing with motion, it is necessary to have a clear definition of the reference system in order to be able to use terms such as position, velocity, speed or acceleration; it could then be stated that motion is the change of position experienced by some bodies with respect to others, so it is stated that motion is relative, since it must always be described with respect to whom or what it describes, as mentioned in [4]. It is necessary to specify some concepts associated with movement: (a) trajectory corresponds to line that the object describes during its motion, which can be rectilinear or curvilinear, the latter in turn can be classified as circular, elliptical or parabolic; (b) displacement corresponds to the directed segment connecting two different positions in the trajectory followed by a body [5]; (c) the distance travelled is the measure of the trajectory; (d) the speed corresponds to the distance travelled per unit time; (e) the velocity is defined as the quotient between the displacement and the elapsed time (it is a vector quantity); finally, (f) the acceleration corresponds to the change in speed experienced by the body per unit time.

All these concepts are articulated when it is stated that a body follows a uniform rectilinear motion when its trajectory describes a straight line maintaining its velocity constant. Various situations in the physical context attempt to solve proposed problems, some of which are posed in everyday language in which a detailed description is given of what happens to the body's movement in certain periods of time; while others resort to the tabular representation of position as a function of time, and others use position-time graphs with the aggravating factor that students must understand the graph in order to interpret it correctly and thus have the ability to extract the data from the problem. The use of graphs has been relevant in the study of physics, for example, space-time graphs were the basis for uniformly accelerated motion, or the study of [6] where he mentions how great physicists use geometric-dynamic diagrams involving a diversity of mathematical concepts to represent the variable and continuous relationship of physical parameters such as time or force.

Derived from the study of physics at the preparatory educational levels for higher education, students are expected to develop a series of competences directly associated with the drawing and interpretation of graphs; as highlighted in a document issued by the “Ministerio de Educación”, Colombia [7], which suggests the analysis of the concepts of kinematics or requests the establishment of relationships between variables associated with movement or highlights the importance of mathematical processes or the use of various registers of representation for the development of competences in physics [7]. On the basis of these guidelines, it could be expected that students reaching secondary technical education grades in Colombia will be able to interpret a kinematics graph in the light of both physical concepts and the mathematical concepts implicit in them, as in the case of the slope of the straight line in a position-time graph that is associated with velocity or the same mathematical concept but in velocity-time graphs to explain the concept of acceleration.

Research on the interpretation of graphs related to uniform rectilinear motion (URM) or accelerated rectilinear motion (ARM) has revealed difficulties in the process of extracting information or in establishing the relationships between the variables involved [8,9]. Thus, [10] is quoted as saying that there are two ways of understanding the graphing process, one is to see it as a technique for sketching the graph of a function, and the other is to identify the meaning and significance of its properties. Then, for this work we assume as a reference for analysis what is stated in [8] who state that interpretation refers to the ability to read a graph both locally and globally to give it meaning or significance, highlighting that “the local refers to the extraction of specific data from the graph and the global refers to the determination of the behaviors and trends of the graphs” [8].

It is from this motivation that this research is developed with the aim of determining the level of information processing in the interpretation of kinematic graphs. It should be noted that the three levels defined by [11] are adopted: (a) elementary, which involves the extraction of data or the isolated reading of points; (b) intermediate, which requires the identification of trends; (c) high, which requires a deep understanding of the behavior of the data; (d) high, which requires a deep understanding of the behavior of the data; (e) low, which requires the identification of trends; and (f) high, which requires a deep understanding of the behavior of the data.

## 2. Method

This research has been organized within the framework of didactic engineering as a research method that is in the register of case studies and whose validation is essentially internal, based on the confrontation between a priori and a posteriori analysis [12].

Four phases can be distinguished: (a) phase 1 preliminary analysis in which a variety of information is analyzed, such as the epistemological analysis of the content, the forms of teaching and their effects, the analysis of the students' conceptions, with the aim of classifying the difficulties according to their epistemological, cognitive or didactic dimension; (b) phase 2, the a priori conception and analysis where the researcher decides to intervene on certain influential variables which in this case correspond to some of didactic origin with the aim of influencing some epistemological ones for which he resorts to the design of didactic sequences; (c) phase 3 experimentation where the intervention process is carried out with the students in which they execute the previously designed didactic sequences; (d) phase 4 of a posteriori analysis in this last stage the process of contrast between the results found in phase 1 and those identified in phase 3 is carried out, in order to carry out the process of internal validation of the pedagogical process carried out.

In this article we report only the results derived from the first phase in which the difficulties present in the students are identified together with the determination of the level of interpretation they possess. For this reason, a qualitative approach is adopted at a cross-sectional descriptive level following a field design [13] since the data were collected directly from students enrolled in the eleventh grade of a private educational institution with emphasis on natural sciences and residing in the metropolitan area of San José de Cúcuta, Colombia, during the month of June 2021 before they went on mid-year holiday.

In total there are 35 students ($N = 35$) and by calculating the sample size ($n_1$) through Equation (1), and by means of Equation (2) a total of 17 students ($n_0 = 17$) are determined as the optimal sample size assuming a probability of success of 55% ($P = 0.55$), an error of 5% ($e = 0.05$) and a confidence level of 95% (equivalent to $Z = 1.96$) with a significance level of 5% ($\alpha = 0.05$). Additionally, it was considered as an inclusion criterion that the students were attending the institution in person, so the sampling process used was probabilistic as stated in [10].

$$n_1 = \frac{N*Z^2*\frac{\alpha}{2}*P*(1-P)}{e^2*(N-1)+Z^2*\frac{\alpha}{2}*P*(1-P)}, \quad (1)$$

$$n_0 = \frac{n_1}{1+\frac{n_1}{N}}. \quad (2)$$

The instrument used incorporates three situations, one is a description of position over time, another is a position-time graph for the URM and the last graph is a velocity-time graph for the ARM. In all cases, three questions are asked, one for each level of understanding defined by [10]. The selection of the situations resulted from an expert panel made up of the researchers, the teacher of the subject at the educational institution and a physics teacher from another educational institution with more than twenty years of experience in the subject, who provided guidance on the type of questions based on the review of three classic school textbooks.

For each situation proposed, open answers are allowed, since the group of students is not so large and, given the purpose of the research, it was of interest to know the opinion of each student on the situations proposed. Subsequently, the responses are analyzed descriptively in terms of their affinities and argumentative intentions.

## 3. Results and discussions

The group of students surveyed is evenly distributed by gender, with ages ranging between 16 years old and 17 years old, with a mean of 16.5 years old and a standard deviation of 0.52, so that generating an interval around the mean of one standard deviation would practically cover all the informants; all of them stated that they had seen physics subjects at the educational institution since they entered sixth grade, but when exploring their liking for the classes given by the teacher, 71.4% said that there are times when the class is not very attractive because it is very routine, the teacher always does the same thing: they mention the subject, a brief explanation, in some cases there is a video and in others they go on to solve exercises based on the formula given by the teacher.

From the characteristics mentioned by the students regarding the methodological approach adopted by the teacher, the teacher has placed special emphasis on solving exercises, neglecting problem solving. Special emphasis is placed on differentiating between exercise and problem as clarified in [14] where it is mentioned that an exercise corresponds to those situations in which the student knows the process to follow to reach the solution, because it has already been worked on by the teacher in class, while a problem requires the student to face a situation that demands the processes of reasoning, analysis, formulation of hypotheses, mathematical modelling and experimentation in search of a possible solution.

As mentioned in the introduction, it is hoped to determine the difficulties that students have in interpreting kinematics graphs in which it can be concluded that, depending on the variables related in the graph, the slope of the straight line corresponds to the velocity or acceleration experienced by a body or object.

### *3.1. Situation 1: description of the trajectory of a body*

Figure 1 corresponds to the trajectory followed by a particle during a period of nine seconds; based on the information provided by the position versus time graph, answer the following questions: (a) what position does the body occupy after the first five seconds? (b) at what interval time does the body stop? (c) what happens to the motion of the body in the interval from 6 to 9 seconds?

When students were asked about the position of the body after the first five seconds, it was found that 42.8% stated that it was only 35 units away from the starting point based on the correspondence between the variables (concept of ordered pair in the cartesian plane $(x, y)$), while 35.7% mentioned that the body had moved from its initial position but their descriptions were ambiguous and therefore did not define the position of the body at the given time, and 21.4% of the remaining students had difficulties in identifying the variables in terms of their respective coordinate axes.

Continuing with the next question, 42.8% of the students' state that two seconds after having started the movement of the body, it stops for two seconds at 20 meters from the starting point, which is reflected in the graph as a horizontal line. The remaining percentage of students' state that the body "does stop" but do not identify the instant of time at which this occurs from the graph; when students are asked to interpret the graph at 6 seconds and 9 seconds, 42.8% of them state that the body returns 45 units of distance to the starting point in those three seconds (using the concept of displacement), while the remaining percentage state that the body is still in motion but offer no description of the variables under study.

From the answers given by the students and in accordance with the levels of interpretation established by [11], it can be affirmed that, in this first situation, 42.8% of the students are at the high level, since they have demonstrated the ability to explain the global behavior of the graph, identifying the characteristics of each section of the trajectory and have associated the graph with the behavior of the variables. The remaining percentage are at the elementary level, characterized by the attempt to explain the behavior of the position as a variable independent of time or time independently of the position, so they are not analyzing the relationship of correspondence that exists between the variables under study.

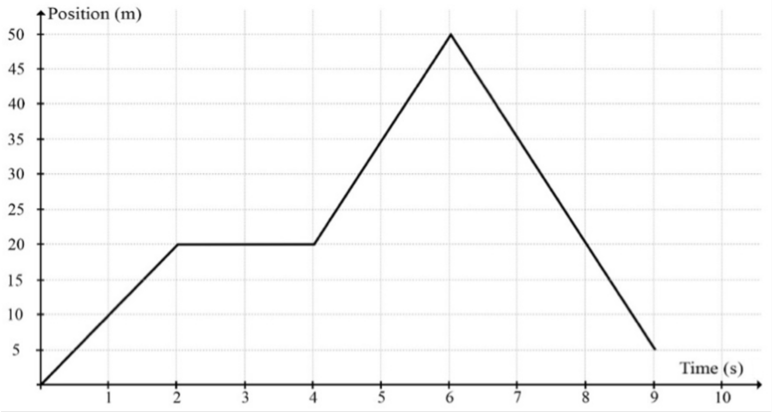

**Figure 1.** Trajectory followed by a particle.

### *3.2. Situation 2: uniform rectilinear motion*

A particle in motion travels a distance $s(t)$ (in metres) in relation to time $t$ (in seconds) as shown in Figure 2. Based on this situation answer the following questions: (a) what is the velocity of the particle at point A? (b) what is the velocity of the particle at point B? (c) can the motion be said to be uniform rectilinear?

When the students were asked about the velocity experienced by the particle at point A and point B, 35.7% of them stated that the velocity is the same at both points, using Equation (3) to calculate the value of the velocity (v), concluding that it increases by 5 meters for every second that elapses since it starts moving. In this group of students, we find statements such as "the speed is the same at all times" or "regardless of the time elapsed, the speed is constant", so these arguments focus them on the high level of interpretation. The remaining percentage is at the elementary level: 35.7% of them try with their arguments to explain the motion of the particle, but without determining the velocity, arguing that they have seen the topic but do not remember the equation kinematic.

$$\mathrm{v} = \frac{\mathrm{s(t)}}{\mathrm{t}}. \tag{3}$$

The remaining percentage (28.6%) explain the behavior of the graph in mathematical terms, stating that the graph represents an increasing function, since the position increases as time goes by, but they do not interpret the slope of the line as representing the particle's speed. The group of elementary level students show a disconnection between the topics developed in physics class and the everyday situations they experience.

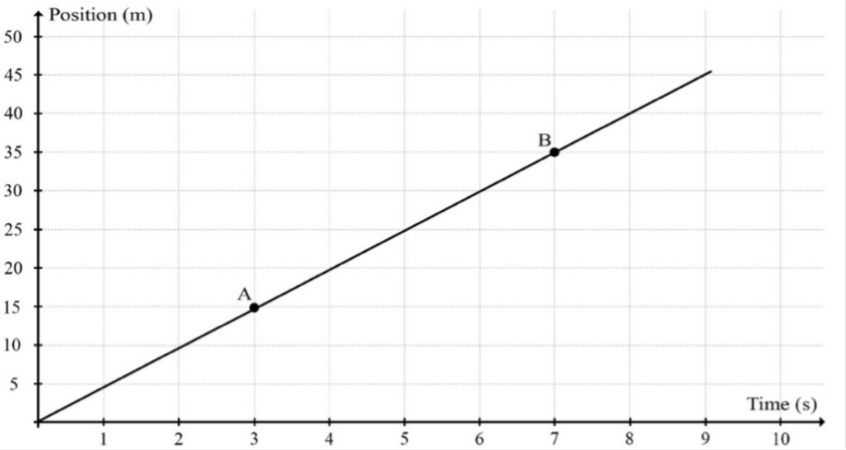

**Figure 2.** Trajectory followed by a particle.

### *3.3. Situation 3. accelerated rectilinear motion*

The following speed-time graph corresponds to the rectilinear motion of a particle; from the information of the graph shown in Figure 3, answer the following questions: (a) how long has the car been in motion? (b) in which section of the trajectory does the driver of the car apply the brakes? (c) in which section does the body experience the greatest acceleration?

Regarding the time that the car has been moving, 71.4% of the students provided correct answers, but with different arguments, for example, answers were found based on the values of the abscissa axis such as during the first 45 seconds, others argued based on the characteristics of the graph, for example, it moves along the entire trajectory analyzed. The remaining percentage (28.6%), present difficulties in the interpretation of the graph since they affirm that the car moves for 30 seconds, since it lasts 15 seconds stopped in section C, which demonstrates the presence of a derivative conception of the position-time graph where, since its trajectory is horizontal, it is assumed that the speed is zero or equivalent to the fact that it stopped, so the students are ignoring the variables associated with the axes of the graph.

Continuing, 64.3% of the students affirm that the brakes are applied in section D and section E, arguing, for example, that the graph descends in its trajectory or that in these sections the relationship is inverse between speed and time. The remaining percentage of students conclude the same, but their argumentation focuses only on the calculation of the differential between the value of the function's ordinate $\Delta\mathrm{v}$ at the end ($\mathrm{v}_{\mathrm{end}} \cong \mathrm{Y}_2$) and at the beginning ($\mathrm{v}_{\mathrm{beginning}} \cong \mathrm{Y}_1$) of the section (Equation (4)), arguing that the values decrease in speed after applying the brakes.

$$\Delta\mathrm{v} = \mathrm{v}_{\mathrm{end}} - \mathrm{v}_{\mathrm{beginning}} \cong \mathrm{Y}_2 - \mathrm{Y}_1. \tag{4}$$

Finally, when asked to identify in which section the acceleration is greater, it was determined that in 78.6% of the cases, the answer was based on the angle of inclination or elevation of the straight line with respect to the horizontal in the section analyzed, which led them to conclude that section B was the steepest, so tacitly the pupils are applying the concept of the slope of the straight line, but they do not support their reasoning with algebraic processes. The rest of the students conclude that in section A and section B the acceleration is positive because the straight line is increasing, while in section D and section E the acceleration is negative because the straight line is decreasing; so, the argumentation provided by the students in this situation places them at the Intermediate level as they try to identify intervals where the velocity and acceleration increase or decrease from the slope of the straight line, but without carrying out any algebraic process.

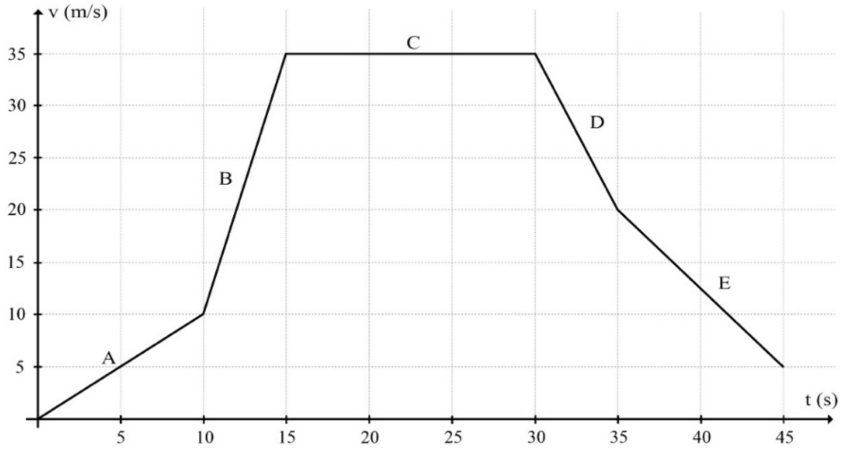

**Figure 3.** Trajectory followed by a particle.

## 4. Conclusion

This work made it possible to determine that the participating students have a series of conceptions about the concepts of both physics and mathematics, which emerge when they are faced with problematic situations that start from the graphic register where processes such as interpretation, reasoning and mathematical modelling are required, the latter becoming a problem since the students showed that they depended on the teacher to provide them with the formula and if they were not provided with it, it was almost impossible for them to solve any situation, despite the concept of slope being the quotient between two magnitudes or two differentials.

The findings derived from this diagnostic process serve as a basis for the design of teaching sequences aimed at improving students' competences in these subjects, *i.e.*, the aim is to ensure that students not only use the formulas, but that they are able, through reasoning, to propose them as the relationship between two known variables, as in the case of the graphs dealt with in this diagnosis. In this way, students can advance to higher levels of understanding of knowledge. These competences are not only necessary in the school curriculum, but are also essential in people's everyday lives, which becomes yet another reason to value the importance of physics and try to understand its concepts, as they contribute to the training of competent citizens.

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