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Evolution of the algebraic error in the evaluation processes mathematics and physics in engineering students

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Abstract. This research aimed to analyze the evolution of the algebraic error in the evaluation processes of mathematics and physics in engineering students, of the Universidad de Pamplona, Villa del Rosario, Colombia. Theoretically, it was based on a typology of errors. The methodology was descriptive quantitative study. The sample consisted of 77 students from engineering programs. The instruments were a Likert scale and a test of knowledge in mathematics and physics involving algebraic arithmetic operations. The results show that 9 errors were replicated in the students of the three subjects evaluated. The most frequent errors were errors when performing algebraic arithmetic operations followed by incorrect own procedures and invalid inferences and errors in the additive resolution of the potency of an unfinished binomial and procedure. In conclusion, it can be said that the same errors appear in the three subjects although it is believed that with the passing of the semesters and the training of the students they should disappear when they finish their academic program.

1. Introduction

At present, engineering professionals trained in mathematics and physics are required, with skills to solve problems, make decisions, understand, develop abstract thinking, seek solutions, and adapt to scientific-technological changes. However, in the university environment there is a stagnation in the evaluation practices remaining, practically, only in the conception of the measurement of knowledge. Then, the evaluation practices are administrative processes that the teacher must validate the knowledge acquired by the student, without considering the errors that appear, which can be the product of previous experiences. In other words, evaluation is not seen as the event that allows exploration, for every wrong answer, to minimize errors, and to become evident opportunities for improvement.

This article presents the error in the evaluation practice not as a deficiency, but as a learning opportunity for the engineering student to understand the process of clearing and making numerical substitutions, equations, identifying variables and identifying formulas through the application of appropriate teaching-learning strategies, which shows that it is possible to minimize the difficulties in the areas of mathematics and physics. In this sense, the objective of the article is to analyze the algebraic error in the evaluation processes of mathematics and physics in engineering students, in a public university.

2. Theoretical framework

2.1. Characterizing aspects of the teaching of algebraic language
With regard to algebraic language, it is stated that the teacher must seek not only the management of operational type symbols and some relationships as in arithmetic, but also expand meaningfully since they are of different nature, in abstract elements that are they are representing through letters [1]. In addition, it is established that algebraic reasoning implies representing, generalizing, and formalizing patterns and regularities in any aspect of mathematics. This type of reasoning is at the heart of mathematics conceived as the science of patterns and order, since it is difficult to find an area of mathematics in which formalizing, and generalizing is not central [2]. Consequently, teachers must build this vision of the central role of algebraic ideas in mathematical and physics activity, and how to develop the teaching of algebraic reasoning across the different levels.

2.2. The evaluation practices in higher education
When thinking about education, the way students' progress from one level to another immediately comes to mind. That is, one of the elements of the education system appears, such as evaluation. The evaluation makes sense to the extent that the effectiveness and the possibility of perfecting the teaching action is tested [3-4]. Likewise, some elements that characterize the evaluation practices, with attention to the university context, must at least start from a consensus among the professors of each specialty on what are the nuclear knowledge that every student should acquire. Consensus that should lead to an analysis of the objectives, methodology and method of evaluation [4-6]. Likewise, the following evaluation characteristics are also referred to: it must be integrated into the design and development of the curriculum, it must be formative, continuous, systematic, criterial, decisive and cooperative [7]. In relation to this, it is proposed that the evaluation merits prior planning, preparation, design and reflection, with all that implies dedication, forecasting of resources, time setting, decision on methodology and strategies, elaboration of instruments, among other aspects [7-8].

2.3. The algebraic error in the evaluation processes
When the error arises in the evaluation processes in the university environment, poorly applied teaching and evaluation strategies that have not changed and that need to be more innovative remain in the classrooms, since the errors have always been far from higher education [9]. Evaluation is the means to support knowledge and the idea is to reach it with success; but if mathematical errors arise, teachers must have enough preparation to understand them [10]. In this sense, when the students present the previous ones or tests in mathematics and physics, the university professors expect the least amount of possible errors. Therefore, it is mentioned that absolute truths are more coveted in university tests in subjects such as mathematics, than errors [11-12].

Regarding the typology of errors in algebraic contexts, the typology presented by García [13] is based on the work of [14-22], which assumes a classification with twelve types of errors: (1) incorrect elimination of denominators, (2) errors in performing arithmetic-algebraic operations, (3) unfinished procedure, (4) incorrect own problems and invalid inferences, (5) partial application of factoring rule by common factor, (6) incorrect association of notable products, (7) use of basic arithmetic ignoring the rules of algebra, (8) error in determining the power of another power, (9) additive resolution of the power of a binomial, (10) incorrect application of the rule of the cube of a binomial, (11) error when making polynomial products and (12) simple calculation error. This classification is explained in detail in the work of Diaz, Hernández and Paz [23].

3. Method
The inquiry that was developed is quantitative, with a descriptive design. The population was made up of students from the academic programs of Mechatronics, Electrical, Telecommunications, Systems, Industrial and Geology Engineering; from the Universidad de Pamplona, Colombia, headquarters of Villa del Rosario, Colombia, in three different semesters. Thus, the first semester consisting of five (5)

courses of the differential calculus subject with 165 students in total was selected. Students were also taken from the third semester of multivariable calculus that has six (6) courses where 150 students are grouped; and students of the fifth semester who were enrolled in the subject of physics were selected, with a single group of 32 students. The sample was of the non-probabilistic type, with participation of 41 students of the first semester, 15 students of the third semester and of the fifth semester 21 students participated.

The technique used to collect the information was the survey. It allowed students to simultaneously respond to a Likert scale on evaluative practices and then respond to a knowledge test about content associated with mathematics and physics involving algebraic arithmetic operations. The Likert scale was organized into 15 questions with five response alternatives whose categories were: totally disagree, disagree, neither agree nor disagree, agree, and totally agree. All this, to measure the student participation dimensions (item 1, 2, 3, 4, 5); relevance of content for life (item 6, 7, 8, 9, 10) and error as a source of learning (item 11, 12, 13, 14, 15).As for the knowledge test, it is necessary to mention that the same battery of questions was prepared for the three semesters that participated in the study. This instrument consists of 16 questions to answer development type. These items were related to the thematic axes of solving equations and systems of linear equations (items 1,2,3,4); application of rules of algebraic operations (items 5,6,7,8); development of notable product rules (items 9,10,11,12); factorization of algebraic expressions (items 13, 14, 15, 16). The test rating scale was 1 to 5, with ratings of 0 for incorrect and 1 for correct.

Since both instruments required a validation process, content validity was used, for which the collaboration of three specialists was requested for the review of the two instruments. Likewise, the reliability that was obtained through a pilot test was applied to 15 students not belonging to the subjects of the population. After the pilot test was applied, the internal consistency of the Likert scale was determined through the Cronbach Alpha Coefficient, which was obtained by replacing the values with the SPPS statistical package which yielded 0.85 reliability, which is located a very high reliability range.

4. Results

After the theoretical and methodological aspects, the results of identifying the errors presented by engineering students are presented. In this regard, first, the results and analysis of the scale of attitude applied to students to review the evaluation practices of teachers are shown. Subsequently, algebraic errors are identified and analyzed in the evaluation processes of mathematics and physics. The Likert scale, on evaluation practices, is presented according to the following dimensions:

- Student participation. To the question if the student participates in the proposal of the contents that he would like to learn? a high percentage of the students responded to totally disagree, it can be said that the pedagogical practices, at present, should encourage the formation of a university student with competences for life. Therefore, the ideas of the students are fundamental [24].
- Relevance of content for life. As for the academic training of the student necessary for life, the participants state that this training is very scarce to develop in the social context. The teaching of mathematical contents, more than any other object of knowledge, should become a relevant element of formation for the student's life. It is important to highlight that the universities of this century must prepare new generations for change and innovation [25]. Thus, classrooms require a dynamic that trains students to understand how knowledge is built.
- The error as a source of learning. It can be observed that when asked, if during the evaluation the teacher considers the student's mistakes to generate strategies? a high percentage of students disagree. These results reflect that the knowledge possessed by the university student very little is considered that the evaluations are elaborated and applied based on the knowledge dictated in class. Some authors agree that prior knowledge is considered essential to acquire new knowledge [26-28]. The acquisition of new knowledge depends to a large degree on the relevant ideas that already exist in the cognitive structure and “the significant learning of human beings occurs through an interaction of new information with the relevant ideas” that already exist in the structure cognitive [29]. When students present tests in mathematics and physics involving

algebraic arithmetic operations, university professors expect the least amount of errors possible. Therefore, it is mentioned that absolute truths are most coveted in university tests in subjects such as mathematics [11]. For its part, it is noted that some university professors have placed special interest in the analysis of errors in the process of building mathematical knowledge [30]. Obviously, these errors influence the learning of the different contents of the student, it is essential that students recognize and assume the need to overcome these obstacles in order to obtain learning achievements. Analyzing errors, especially towards problem solving, allows teachers to organize strategies for better learning.

According to the knowledge test applied to the students, in the different subjects, the following errors were found in accordance with the proposed García (Table 1) [13].

Table 1. Frequencies of errors that arise in each semester and subject.

Type of error	Differential calculus (first semester)	Multivariable calculus (third semester)	Physics (fifth semester)
Incorrect elimination of denominators	2.3%	1.2%	4.4%
Errors when performing arithmetic and algebraic operations	27.2%	28%	24.2%
Unfinished procedure	9.2%	14.6%	7.6%
Incorrect own procedures and invalid inferences	30.6%	18.3%	32.5%
Partial application of factoring rule by common factor	8.6%	8.5%	6.4%
Incorrect association of notable products	6.4%	7.3%	8.3%
Use of basic arithmetic ignoring the rules of algebra	5.2%	8.5%	7.6%
Error in determining the power of another power	0.0%	0.0%	0.0%
Additive resolution of the power of a binomial	4.5%	13.2%	3.8%
Incorrect application of the binomial cube rule	0.0%	0.0%	0.0%
Error when making polynomial products	6.9%	0.0%	5.1%
Simple calculation error	0.0%	0.0%	0.0%

Table 1 shows the errors that arise in each semester and subject. It should be noted that errors appear, in the three semesters, contrary to what some teachers think, when they state that algebraic errors disappear in the last semesters. The errors that were not found in the tests correspond to the determination of the power of another power, incorrect application of the rule of the cube of a binomial and simple calculation. Those with the greatest appearance in the sample were errors when performing arithmetic and algebraic operations and incorrect procedures and invalid inferences. Likewise, the results of the attitude scale show a low use of the resources that students possess, as their previous knowledge, and a low use of error as a source of learning. With respect to the theoretical assumptions, it is mentioned that the contents mathematics imply a teaching and evaluation process that allow observing the performance in mathematics necessary to raise the index of grades in said subject [2]. Teachers in their evaluation processes must help students in the mistakes they make so that a different vision of the teaching of mathematics in higher education can be constructed [31]. Most professors at the university level teach mathematics in a routine, expository and tedious manner [32]. Therefore, the teaching of mathematics in the university they must be implemented in the classroom of activities that make sense for the student which allows him to improve performance in this area of knowledge [33].

To reflect on the teaching of mathematical and physics contents is to think of activities that allow promoting the development of the student for his professional practice. Therefore, the teaching of these disciplines should be done from different perspectives; it is thought that the teaching of subjects associated with mathematics and physics in the university should start from a dialogic path that interrelates some didactic elements with the student's knowledge. In such a way, that mathematical knowledge constitutes a factor of adaptation of the student to the university, from the realization of diverse daily activities. For this reason, it is mentioned that the study of the way in which the contents associated with mathematical subjects are originated and conceptualized by the student continues to be the subject of analysis for mathematicians [33].

When reviewing the results, it is appreciated with respect to the items where a high percentage of errors can be detected in the three semesters evaluated and that are related to Garcia's proposal [13]. In this sense, it can be inferred that in this study 6 of the 12 errors proposed by the mentioned author are replicated in the first semester; in the third semester, 8 of the errors are repeated and in the fifth semester 9 errors are replicated. Therefore, in the three semesters, the errors (common to the three) of greatest occurrence were errors when performing arithmetic-algebraic operations, incorrect own procedures and invalid inferences and unfinished procedure.

5. Conclusions

Regarding the diagnosis of the errors that appear in the students in the subjects of mathematics and physics of the Engineering programs, the typology of errors raised in the theoretical framework was found. In the differential calculus subject the students replicated 9 errors, the most appearing were the errors when performing algebraic arithmetic operations and the incorrect procedures and invalid inferences. In the vector calculus subject, 8 errors were found, the most frequent being the errors when performing algebraic arithmetic operations followed by incorrect own procedures and invalid inferences and errors in the additive resolution of the power of a binomial. Finally, 9 types of errors arose in the subject of physics. The error with the highest percentage of occurrence was the so-called own and incorrect procedures and invalid inferences, then the errors appear when performing algebraic arithmetic operations and an unfinished procedure. It can be inferred that contrary to what was thought at the beginning of the study, that as students' progress in the semesters, algebraic errors in mathematics decrease, it was not so, because in the three semesters analyzed common errors emerged.

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