

# Mathematical processes and pedagogic practice: Characterization of the teachers in basic and middle education

## Procesos matemáticos y práctica pedagógica: caracterización de los docentes de educación básica y media

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#### ABSTRACT:

This report is generated from a sample of 31 professionals working as math teachers in the basic and middle education levels on six different private institutions located in the urban area of San José de Cúcuta, Colombia. An instrument that takes as reference the five mathematical processes mentioned on the Principles and Standards for the Mathematical Education of the National Council of Teachers of Mathematics – NCTM – 2003, in total 36 items, was applied. Derived from the processing of the recollected data, the poor establishment of connections with other areas of knowledge is identified as the main weakness in the pedagogic practice of the educator. Reflections about the consequences of the impact that it can have on the teaching process appear: Mathematics are shown as an isolated knowledge, disconnected from the educative system and the everyday. It concludes in that, in one way or another, affects the motivation of the student in the learning of mathematics. Actions are proposed to encourage the connections with more areas in the teaching practice.

**Keywords:** Mathematics, NCTM, Mathematical Processes, Solving problems

#### RESUMEN:

Esta investigación se genera de una muestra de 31 profesionales que trabajan como docentes de matemáticas en los niveles de la Educación Básica en seis instituciones privadas en el casco urbano de San José de Cúcuta – Colombia. El instrumento aplicado toma como referencia los cinco procesos matemáticos mencionados en los Principios y Estándares para la Educación matemática definidos por el Consejo Nacional de Profesores de Matemáticas – Estados Unidos, con un total de 36 ítems. Se identifica, derivado del tratamiento de los datos recogidos, la principal debilidad en la práctica pedagógica de los profesores, el escaso establecimiento de conexiones con otras áreas del conocimiento. Se concluye que esto, de una manera u otra, afecta la motivación del estudiante en el aprendizaje de las matemáticas.

**Palabras clave:** Matemáticas, NCTM, Procesos matemáticos, resolución de problemas.

## 1. Introduction

Many nations in the world have recognized that education is one of the factors with the greatest impact on the progress of society and on the people themselves. In Colombia at the beginning of the 20th century, it was thought that primary education provided the necessary resources to get ahead, but with the passing of time society has been increasing the demand in the preparation of the population, resulting in people wanting to achieve an adequate level of formation to guarantee greater job opportunities with better salary income. A better education manages to potentiate the intellectual development of a country through research practices whose products could reduce poverty, guarantee a better quality of life, give greater access to technological advances and increase the level of cultural tolerance. For this to be fulfilled, the quality of the pedagogical practices used by teachers in the classroom must be improved to assure the conceptual appropriation of knowledge and its application in the solution of problems demanded by the context in which the student is involved (Delgado, 2014), (Gamboa-Suárez, 2016).

In order to reach such aim, it is necessary to rethink the diverse aspects associated with the educative act, for example, as affirmed by Duque, Vallejo & Rodríguez (2016) the classroom must be thought of not a simple space, but a place where the master joins in an area of production with his students, turning said place into one where the investigation and the knowledge flows in a way that aims at the attitudes of those who are participating, helping in the development of the people.

On the other hand, the pedagogical practice of the teacher must be thought as the actions and insights made in the classroom. With this in mind there are many aspects to consider: how the class is prepared and how the teacher carries it out, what pedagogical meditations are implemented, what experiences and joint contents are present in the class, how is the evaluation process conceived and executed and what process is generated from the results of the evaluation... in short, everything that has been constituted in the curriculum of the schools (Gómez, 2010). It is necessary to add to the formation of the student, the abilities to manage the diverse knowledge of its vital, affective and social necessities and the outside of the educative institutions. Lately these have been the motivations of the investigations in Didactic and Pedagogical Sciences, everything with the clear intention of channeling the education using more efficient courses of action (Hunt, 2009), (Raczynski & Muñoz, 2005).

The present education requires to give a meaning to the formation process, which becomes impossible to reach in the particular case of the Mathematics if it continues being developed from a traditional approach of education that has left one long list of academic difficulties in the process of conceptual apprehension. In Santaolalla (2009) the urgent necessity of changes in the education of Mathematics on all the educative levels but in the initial degrees is emphasized. Fundamentally, there have been two main reasons: first, the great development of some specific didactics have revealed that the way in which some knowledges are learned varies depending on the content itself, because they imply a diverse cognitive processing and the exercise supposes specialized abilities; the second reason is related with the attention that has been dedicated to the education and learning the Mathematics based on its importance in the scholastic formation like, paradoxically, the high indices of failure that are frequently exhibited by the students in the internal or external tests in Colombia.

The advances that have been made in the special didactics of mathematics, however, have not produced all the desirable results in student achievement. In fact, the number of failed of mediocre students continues to dominate the national scenario and that of other Latin American countries (Casassus, Cusato, Froemel & Palafox, 2001), (Hernández-Suárez, Prada-Núñez & Gamboa-Suárez, 2017). The causes of these nefarious results could be diverse, but given the orientation of this research, the attention will focus on analysing the mathematical processes that take place in the classroom, since a criticism that is made to the process of teaching Mathematics has been the excessive instrumentalization of knowledge in the school environment, too often reduced to a clearly operative, mechanical and repetitive expression that considerably limits the correct understanding of concepts. In the classroom, the student is frequently presented with a series of exercises whose resolution involves the instrumental replication of a procedure. This practice produces in some cases correct answers, but rarely understanding of the processes and concepts. Such instrumentalization of Mathematics has disadvantaged student's critical attitude towards the resolution of evaluation activities (Barrera-Osorio, Maldonado & Rodríguez, 2012) and a lack of interest in the development of problem-solving skills, an aspect that is exacerbated with the loss of sensitivity to the frequent error that, for the same reasons, students exhibit when operating an exercise or problem.

### 1.1. Importance of the Mathematical Processes

As stated by the National Council of Mathematics Teachers – NCTM (2013) it is shown that the process of teaching Mathematics should be oriented to a conceptual understanding that can only be achieved by mathematical reasoning, implying to play down the importance of learning procedures without any connection. To guarantee the fulfilment of this purpose, the NCTM proposes five mathematical processes that must be developed in the student:

- a) *Resolution of problems*, which should lead the student to the construction of new mathematical knowledge in diverse contexts of application.
- b) *Reasoning and test*, requires students to investigate, propose and evaluate various conjectures in such a way that the results obtained propose logical arguments supported by different types of reasoning.
- c) *Communication*, the student is required to dominate and coherently articulate various types of representations of mathematical concepts.
- d) *Connections*, it is expected that the student will understand how mathematical ideas are related and articulated in a coherent whole, while they can be applied in diverse non-mathematical contexts.
- e) *Representations* support the processes of modelling and interpretation of phenomena in various fields as a field of application in Mathematics.

## 2. Methodology

### 2.1. Focus and design

The research is framed in the quantitative approach at the descriptive level of cross-cutting and fieldwork: (a) quantitative because it relies on descriptive statistics to set out the various characteristics observed in informants; (b) cross-sectional because it is capturing the opinion of informants at a specific time in order to investigate a particular issue; and (c) fieldwork because the data is collected directly from the primary source.

### 2.2. Population and sample

The population was made up of the docents that worked in six private institutions located in the urban area of the city San José de Cúcuta. For the selection of the sample, the orientation of the subject of Mathematics in the participating institutions was considered as the only type of inclusion. In this way, a group of 32 teachers working in the various grades of Primary and/or Secondary Basis Education were consolidated; this complements the report that was conducted with a group of 105 teachers from the public institutions of the Norte de Santander Department.

### 2.3. Instrument for data collection

The instrument used consists of two sections that are detailed in Table 1. The first section corresponds to the categories associated with the affective domain, while the second section lists the indicators associated with the mathematical processes defined by the NCTM.

**Table 1**  
Composition of the instrument

| Section | Categories             | Sub-categories                | Number of items |
|---------|------------------------|-------------------------------|-----------------|
| 1       | Affective Domain       | Beliefs about mathematics     | 9               |
|         |                        | Attitudes towards Mathematics | 10              |
| 2       | Mathematical Processes | Problem solving               | 7               |
|         |                        | Reasoning and Test            | 7               |
|         |                        | Connections                   | 8               |
|         |                        | Communication                 | 7               |
|         |                        | Representation                | 7               |

The section of the instrument concerning the categories of the affective domain has been analysed in detail in the work of Fernández, Hernández, Prada & Ramírez (2018). This report is dedicated exclusively to the analysis of the mathematical processes that have been rated by teachers using a Likert scale with five response levels, which present two negative (grades 1 and 2), one neutral (grade 3) and two positive (grades 4 and 5). The group of respondents presents a majority of masculine gender, whose ages range between 22 and 55 years. Regarding their professional training 16% of the respondents are engineers and the remaining percentage, graduates in Mathematics. Approximately 52% of them have 11 years or less of experience as a teacher in the various grades of Basic Education.

## 3. Results

Table 2 allows us to compare the different mathematical processes grouped into three performance levels obtained as follows: Very Disagree and Disagree responses generated the level of Negative performance, while Very Agree and Agree responses generated the Positive performance level. This adjustment is made only in order to improve the interpretation of the results. The results show that in the opinion of the teachers surveyed, the Connections process is the one that has been promoted to a lesser extent among their students, while all the others are shown as strength. The presence of these mathematical processes in the formation of students contributes to the development of logical thinking and an adequate quantitative performance.

**Table 2**  
Percentage of average performance of teachers according to mathematical process

| Mathematical Process | Performance Level |         |          |
|----------------------|-------------------|---------|----------|
|                      | Negative          | Neutral | Positive |
| Problem Solving      | 0.0%              | 7.4%    | 92.6%    |
| Reasoning and Test   | 0.0%              | 6.9%    | 93.1%    |
| Connections          | 10.5%             | 19.4%   | 70.2%    |
| Communication        | 0.0%              | 9.7%    | 90.3%    |
| Representation       | 0.0%              | 8.3%    | 91.7%    |

Given that the Connections process is identified as a weakness in the pedagogical practice developed by teachers, it demands special attention since showing the student the relationship between mathematical concepts and the totality of daily activities would help to awaken the interest in the study and learning of these concepts.

Table 3 presents the detailed analysis of the various aspects evaluated in the Problem Resolution process. As can be seen in the opinion of the teachers surveyed, in their teaching practice, situations that originate from the student's daily life are always proposed, resorting to the use of concrete material or other resources that facilitate the understanding of the scenario raised, which is complemented by the presentation of many and diverse circumstances on the same concept addressed. In order to enhance this mathematical process, it is necessary to see the question as a teaching resource that favours the understanding of the different situations since by means of it one can guide the reasoning around the situation and the construction of possible paths of solution (Días & Poblete, 2001), (Zuleta, 2005).

**Table 3**  
Percentage of teachers' performance  
in the Problem-Solving process

| Problem Solving  | Performance level |         |          |
|--|-------------------|---------|----------|
|  | Negative          | Neutral | Positive |
| You raise problematic situations using different types of support (oral, with parallel analogies, with manipulee or concrete material to work on, pictorial material). | 0.0%              | 9.7%    | 90.3%    |
| Contextualizes the problematic situations to the daily life of the students.   | 0.0%              | 0.0%    | 100.0%   |
| It proposes problematic situations of diverse types on the same mathematical concept.  | 0.0%              | 3.2%    | 96.8%    |
| Ask questions that generate research and exploration to solve the problem.   | 0.0%              | 9.7%    | 90.3%    |
| It allows the student to use concrete and / or pictorial material with oral support to work on solving problems.   | 0.0%              | 9.7%    | 90.3%    |
| It keeps the student engaged and actively participating in the problem-solving process.  | 0.0%              | 9.7%    | 90.3%    |
| Promotes discussion around problem-solving strategies and results.   | 0.0%              | 9.9%    | 90.3%    |

Table 4 describes the aspects considered in the Reasoning and Testing process. It is highlighted that the totality of the respondents affirm that they use questions to help their students to discuss their answers, which turns out to be a contradiction with what was identified in the previous process, where at least 10% affirm that they do not use the question as a teaching resource. On the other hand, the teachers affirm that they allow their students to discover, analyse and propose different ways of resolution with the respective discussion using the support of mathematical reasoning and the use of concrete material in order to clarify the presented doubts (Duval, 2006).

**Table 4**  
Percentage of teachers' performance  
in the Reasoning and Testing process

| Reasoning and Test   | Performance level |         |          |
|--|-------------------|---------|----------|
|  | Negative          | Neutral | Positive |
| Invite students to make their own guesses, using, for example, trial and error.                        | 0.0%              | 9.7%    | 90.3%    |
| It allows the students themselves to discover, analyse and propose different means of resolution.      | 0.0%              | 3.2%    | 96.8%    |
| Ask the students to explain, justify or argue the strategies or techniques used during the resolution. | 0.0%              | 6.5%    | 93.5%    |
| Ask questions to help students argue their answers.  | 0.0%              | 0.0%    | 100.0%   |
| Promotes students to check conjectures of everyday life.   | 0.0%              | 12.9%   | 87.1%    |
| Promotes the support of mathematical reasoning.  | 0.0%              | 9.7%    | 90.3%    |
| Make and deliver feedback with concrete manipulative material.   | 0.0%              | 6.5%    | 93.5%    |

Table 5 details the opinion of the assessed teachers in the aspects evaluated in the Connections process, by mean of which all of them show that in their pedagogical practice they make connections between different mathematical contents but relations with mathematical concepts with other areas such as music, literature, artistic expressions or other means of psychomotor expression are very little worked.

**Table 5**  
Percentage of student performance  
in the Connections process

| Connections  | Performance level |         |          |
|--|-------------------|---------|----------|
|  | Negative          | Neutral | Positive |
| Consider the students' everyday mathematical experiences to move toward more formal mathematics. | 0.0%              | 3.2%    | 96.7%    |
| Make connections between different mathematical contents.  | 0.0%              | 0.0%    | 100.0%   |

|   |       |       |       |
|---|-------|-------|-------|
| It develops mathematical activities linked to musical contexts.   | 29.1% | 38.7% | 32.3% |
| Works mathematics linking them with children's literature.  | 22.6% | 32.3% | 45.2% |
| It relates mathematics to artistic expression.  | 13.0% | 29.0% | 58.1% |
| Generates mathematical knowledge through contexts linked to psychomotricity.                                | 16.1% | 25.8% | 58.1% |
| Promotes students to apply mathematical knowledge to everyday situations.                                   | 0.0%  | 3.2%  | 96.8% |
| Promotes students to apply mathematical knowledge in the context of nature, dealing with natural phenomena. | 3.2%  | 22.6% | 74.2% |

Regarding the processes of Communication and Representation, they are closely related and it is highlighted that in the opinion of the teachers surveyed, they state that in their classes they make use of various types of representations associated with mathematical concepts and encourage spaces for discussion and articulation between these types of representation, promoting with these activities the respect for the opinion of the other and emphasising that the only way to win a discussion is through the presentation of valid arguments supported by logical reasoning processes.

## 4. Conclusions

When reviewing the presence of mathematical processes considered by the NCTM in the pedagogical practice of a group of teachers working in private institutions with excellent performance in standardized tests, it can be concluded that the five processes are developed and enhanced in the classroom with special emphasis in Reasoning, Representation and Problem Solving as a requirement of academic promotion. The process of Communication is assumed as a commitment within the integral formation in all the areas and definitively the Connections of the mathematical concepts with other areas is evidenced as a weakness in the formation process.

It is worth noting that this is the opinion of the teacher, then it is recommended for future research to investigate the opinion of the student and the area coordinator in each institution in order to triangulate the information and validate the results.

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