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Abstract. Economic growth is a function of the interactions between the different productive factors framed in the economic policy of an economy. The present work tries to explain the economic growth in developing economies, for which a variation of the model on the dynamics of growth proposed by Lukas is proposed, consisting of using, instead of the production function of Cobb-Douglas, a function of production with constant substitution elasticity, since it is very probable that in incipient economies, this one better reflects the functional relationship between factors of production.

1. Introduction

Economic growth is understood as a dynamic process in which each economy establishes interactions between the different productive factors that, together with economic policies, allows the generation of a greater quantity of goods and services produced, which improves the wellbeing of the population. [1,2]. A model of economic growth is based on economic theory to establish basic fundamental assumptions that allow proposing an interaction between the factors of production in order to explain the determinants of economic growth [3,4]

Based on exogenous and endogenous growth theories, different models have been proposed to explain economic growth including the human factor, capital accumulation and technological change among other factors of production. In this sense, Paul Romer [5,6], Lucas [7], Aghion and Howitt [8], Grosman and Hellpman [9], Guellec and Ralle [10] and Gaviria [11] among others, present theoretical works that explain endogenous growth.

Lucas [7] presents three models of economic growth to describe the production of a country based on its levels of physical and human capital and its level of technological acquis, using a production function of Cobb-Douglas. In the first model emphasizes the accumulation of human capital and technological change as determinants of economic growth, considers, like Solow, that the rate of change of technology is exogenous and determines the trajectories that per capita consumption should follow (control variable) and the stock of capital (state variable) in order to maximize utility over time.

In the second model, it assumes that technological change is endogenous and uses the approaches of Usawa [12] to measure its rate of growth. Its purpose is to find the trajectories that must be followed by the variables of control, per capita consumption and effort destined to production, as well as the variables of state, level of knowledge and stock of capital, to maximize the function of intertemporal utility. Finds that the growth of capital must be equal to the sum of the growth of the population and technological stocks. The third model emphasizes international trade and the accumulation of human capital through learning in action as engines of economic growth.

But nevertheless; even when Lucas [7] states that in developing countries the elasticity of substitution between factors of production is less than 1, he develops his work based on Cobb-Douglas production functions which assume an elasticity of substitution equal to 1 In this paper we propose a production function with constant substitution elasticity since, in countries with emerging economies, it is very likely that it is less than unity.

2. Economic growth model with exogenous technological change

Like Lucas [7], it is considered a closed economy, with competitive markets, with identical rational agents, endowed with a technology with constant returns to scale. Let $L(t)$ be the number of people (or, equivalently, the number of man hours) willing to produce in a period of time t, with growth rate $\eta = \dot{L}(t)/L(t)$ given exogenously. Per capita consumption $c(t)$ is a flow of units of a particular good. The preferences on the flow of per capita consumption are measured by the intertemporal utility function defined in Equation 1, in which $\mathcal{U}$ is a function that depends only on the level of per capita consumption over time and ρ is the discount rate. This rate is expected to be high (higher than the interest rate) in the emerging economies since in them the present consumption is preferred to the future, while in developed economies its value is low (lower than the interest rate).

$$\int_0^\infty \mathcal{U}[c(t)]e^{-\rho t}dt = \int_0^\infty \frac{c(t)^{1-\sigma}}{1-\sigma}e^{-\rho t}dt \quad (1)$$

In Equation (1) we include a utility function with elasticity of substitution $(1/\sigma)$ between present consumption and constant future consumption, which allows us to describe the preferences among them. If σ is large, there is low elasticity between present and future consumption, this implies great preference for present consumption and low response of savings to the interest rate with high aversion to risk, which is to be expected in poor economies.

The production in a period t, is determined by the level of technology, A(t), present in that period and by the levels of capital, K(t), and work, L(t), used in that period. Equation (2) describes the level of production, based on the variables described, using a production function with constant substitution elasticity, in which the relative share of each factor in the final product is determined by the distribution parameter ∂ $(0 \leq \delta \leq 1)$ and the elasticity of substitution in production is given by $\sigma_p = 1/(1+\alpha)$, where α is the substitution parameter $(\alpha > -1, \alpha \neq 0)$.

$$Y(t) = A(t)[\delta K(t)^{-\alpha} + (1-\delta)L(t)^{-\alpha}]^{-1/\alpha} \quad (2)$$

The elasticity of substitution is a local measure of substitution is a local measure of the substitution between the factors of production, that is, for a given level of production, measures the proportional change in the use of factors as a result of a proportional change in the marginal rate of labor substitution by capital (TMS), $\sigma_p = [TMS/(K/L)]/[\partial TMS/\partial(K/L)]$; it can also be understood as a measure of the percentage change in the rate of use of the factors of production as a result of a percentage change in the relative prices of these factors. Thus, for high values of σ_p, the TMS responds intensely to changes in the relative prices of the factors and vice versa [13].

In this model it is considered that the technological change is given exogenously, then its growth rate $\mu = \dot{A}(t)/A(t)$ is given exogenously and the function that describes the trajectory of the technology over time is of the exponential type $A(t) = A_0e^{\mu t}$. It is also assumed that the per capita production of the only good is divided into consumption and capital accumulation. Thus, the net national income is given by: $Y(t) = c(t)L(t) + \dot{K}(t)$ where $\dot{K}(t)$ is the net investment [$\dot{K}(t)/K(t)$ is the rate of change over time of the stock of capital] and since $c(t)$ represents per capita consumption, the expression $c(t)L(t)$ represents the level of national consumption.

The problem to solve consists of determining the trajectory in the time of the percapita consumption $c(t)$, that maximizes the function of utility given by the Equation (1) and Equation (2) incorporating the expression deduced on the income, which implies to solve the problem of control (Equation (3) and Equation (4)), in which $c(t)$ is the control variable and $K(t)$ is the state variable:

$$\text{Max} \int_0^\infty \frac{c(t)^{1-\sigma}}{1-\sigma} e^{-\rho t} dt \tag{3}$$

$$\text{s.a.}: c(t)L(t) + \dot{K}(t) = A(t)[\delta K(t)^{-\alpha} + (1-\delta)L(t)^{-\alpha}]^{-1/\alpha} \tag{4}$$

The solution to the corresponding Hamiltonian is obtained by applying the principle of maximization of Pontriagyn [13] which requires for an optimal solution, the satisfaction of the following necessary conditions: [i] $c(t)^{-\sigma} = \lambda e^{\rho t} = \theta(t)$, that is, that in the margin, the goods must be equally appreciated in their uses: consumption and capital accumulation. [ii] $\dot{\theta}(t)/\theta(t) = \rho + \dot{\lambda}/\lambda = -\sigma\, \dot{c}(t)/c(t)$, per capita consumption must vary inversely and proportional to the efficiency price of capital with proportionality constant the elasticity of substitution of consumption present for future consumption. [iii] $\dot{\theta}(t)/\theta(t) = \rho - \frac{\delta}{A(t)^\alpha} \varphi(t)^{\alpha+1}$ the shadow price change rate or The efficiency of capital used to increase physical capital must be equal to the discount rate minus the marginal productivity of capital. [iv] $\dot{\varphi}(t)/\varphi(t) = \frac{\alpha}{1+\alpha}\mu = (1-\sigma_p)\mu$ the rate of change of the product to capital ratio is constant and proportional to the rate of change of technology, where the constant of proportionality is $(1-\sigma_p)$; therefore, the optimal trajectory of the product to capital ratio is of the exponential type: $\varphi(t) = \varphi_0 e^{(1-\sigma_p)\mu t}$.

It should be noted here that both the magnitude and the sign of the rate of change of the product to capital ratio depends on the value of the elasticity of substitution in production. Thus, if $\sigma_p = 1$, the product to capital ratio is constant over time since its exchange rate is zero (this is the case of a production function type Cobb-Douglas, treated by Lukas). If there is substitution in production, $\sigma_p > 1$, the product to capital ratio will be decreasing and the rate at which it will decrease will be greater the greater the elasticity of substitution. If there is no substitution in production, or if the substitution is very low, $0 < \sigma_p < 1$, the product to capital ratio will be increasing and its growth rate will be higher as the elasticity of substitution becomes lower.

To find the optimal trajectory of capital and its exchange rate, we start from the production function and, once found, we analyse the restrictions that must be imposed on the growth rate of per capita consumption to satisfy the optimization conditions. So, the rate of growth of capital is given by $\frac{\dot{K}(t)}{K(t)} = \eta + \frac{A(t)^\alpha[\mu - \dot{\varphi}(t)/\varphi(t)]}{A(t)^\alpha - \delta\varphi(t)^\alpha}$, which implies that the variation of capital is Equation (5):

$$\frac{\dot{K}(t)}{K(t)} = \eta + \frac{\mu\sigma_p {A_o}^\alpha}{{A_o}^\alpha - \delta{\varphi_0}^\alpha e^{-\alpha\sigma_p\mu t}} \tag{5}$$

The growth rate of the economy is obtained from $G = \frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{K}(t)}{K(t)} + \frac{\dot{\varphi}(t)}{\varphi(t)}$ and it is expressed as follows:

$$G = (1-\sigma_p)\mu + \eta + \frac{\mu\sigma_p {A_o}^\alpha}{{A_o}^\alpha - \delta{\varphi_0}^\alpha e^{-\alpha\sigma_p\mu t}} \tag{6}$$

Note that these rates (Equation (5) and Equation (6)) are constant only in the case that the elasticity of substitution in production is 1.

Now, to analyse the behaviour in the long term, and therefore the stability of the model, we calculate its derivative with respect to time, which can be expressed as $\frac{\partial G}{\partial t} = \frac{\partial[\dot{K}(t)/K(t)]}{\partial t} = \frac{-\alpha\delta\sigma_p{}^2 A_0{}^\alpha \varphi_0{}^\alpha e^{-\alpha\sigma p \mu t}}{[A_0{}^\alpha - \delta\varphi_0{}^\alpha e^{-\alpha\sigma p \mu t}]^2}$. Note that the sign of the derivative depends solely on the substitution parameter α. Thus it is obtained that if $\alpha \to 0$ (that is, when the elasticity of substitution is 1) the rates of variation of capital and of production are constant over time. If there is substitution in production $(\sigma_p > 1;\ -1 < \alpha < 0)$ the rates of variation of capital and product are increasing over time. If there is no substitution in production $(0 < \sigma_p < 1;\ \alpha > 0)$ the rates of variation of capital and product are decreasing over time and tend to be zero in the long term.

It is also convenient to observe the behaviour of the capital-labour relationship over time, for this, we start from the production function and, after some algebraic transformations we obtain $\frac{K(t)}{L(t)} = (1-\delta)^{-1/\alpha}\varphi(t)^{-1}[A(t)^\alpha - \delta\varphi(t)^\alpha]^{1/\alpha}$, whose derivative with respect to time is $\frac{\partial[K(t)/L(t)]}{\partial t} = \mu\sigma_p(1-\delta)^{-1/\alpha}\varphi(t)^{-1}A(t)^\alpha[A(t)^\alpha - \delta\varphi(t)^\alpha]^{(1-\alpha)/\alpha}$. This derivative is positive for any value of $t > 0$ regardless of the value that the substitution parameter α takes; which indicates that over time, following the optimal trajectory, the capital-labor relationship shows a continuous growth.

The previous discussion implies that three different cases must be considered (depending on the elasticity of substitution) in order to determine the rate at which per capita consumption must grow in order to satisfy the differential Equation $\dot{K}(t) = K(t)\varphi(t) - c(t)L(t)$ and stability of the model is achieved in the long term.

First, if the substitution parameter tends to be zero $(\alpha \to 0)$, the CES production function tends to be a Cobb-Douglas type production function with substitution elasticity equal to 1 $(\sigma_p = 1)$ and in this case the product to capital ratio will be constant over time. When the substitution parameter is zero, we have that $G = \frac{\dot{K}(t)}{K(t)} = \eta + \frac{\mu}{1-\delta}$, which implies that the stock of capital and the national product they grow at the same constant rate and therefore the product to capital ratio will remain constant over time. This result is equivalent to that obtained by Lucas [7] using a Cobb-Douglas production function with exogenous technological change.

The cases of interest in this investigation are those in which $\alpha \neq 0$, since the results thus obtained will differ from those obtained by Lucas, since if $\alpha \neq 0$ can be considered economies with substitution elasticity in the production different from 1. It is quite possible that in emerging economies there will be a fairly low substitution in production, but high levels of substitution could be expected in developed economies. However, this situation does not occur in developed economies, perhaps because their growth depends more on the technological level than on capital.

The second case to consider is when $-1 < \alpha < 0$ in which the elasticity of substitution in production is greater than unity $\left(\sigma_p = \frac{1}{1+\alpha} > 1\right)$, we obtain that the exchange rate of the product to capital ratio $\frac{\dot{\varphi}(t)}{\varphi(t)} = \frac{\alpha}{1+\alpha}\mu = (1-\sigma_p)\mu$ is negative and therefore it can be stated that this relationship decreases over time to such an extent that in the long term (when $t \to \infty$) the product to capital ratio tends to be zero, indicating that in the long term high levels of capital are required to achieve low levels of production. Likewise, when t reaches very large values, by Equation 5 it is observed that capital tends to grow at a constant rate η (capital tends to grow at the same rate as the labor force) and by Equation 6 it is observed that the production of the economy tends to grow at the constant rate $\eta + (1-\sigma_p)\mu$. It should be noted that because $1 - \sigma_p < 0$, the economy grows at a rate lower than the population growth rate. In this case, the consumption growth rate $g = \frac{\dot{c}(t)}{c(t)} = \frac{\dot{L}(t)}{L(t)} - \frac{\dot{K}(t)}{K(t)}$ in the short term it is changing, but for large values of t its value tends to be zero, which implies that in the long term per capita consumption tends to be constant.

In the third case we have $\alpha > 1$, that is, the elasticity of substitution in production is less than 1 $(\sigma_p < 1)$, which is a very interesting case since the developing countries have an elasticity of Substitution in production is less than unity, given that they have a lower labor share in production than developed countries. Under these circumstances, the exchange rate of the product to capital ratio is positive, indicating that the product-capital ratio is increasing as time passes.

Similarly, for large values of t, capital growth tends to stabilize around the value $\eta + \mu\sigma_p$ indicating that capital must grow at a higher rate than the population; Also, the growth rate of the economy tends to be close to $\eta + \mu$, indicating that economic growth is regulated by population growth and technology growth. The rate of growth of per capita consumption in the first years undergoes some variations, but with the passing of time it stabilizes around the rate of growth of the population; Thus, even when the population increases, economic growth will allow an increase in per capita consumption equal to population growth.

Note that when the elasticity of substitution is less than unity, the growth rate of capital is decreasing and tends to stabilize in the long term. If this were not the case and the rate of growth of capital would grow indefinitely, in developing economies there would be an unemployment problem since in these economies there is not enough saving to hook workers at certain prices, prices would be too high to the level of existing savings. On the other hand, to analyse the effect that the level of elasticity of substitution in production can cause in saving, we write the saving rate in the following way $s = \frac{\dot{K}(t)/K(t)}{\dot{Y}(t)/K(t)} = \frac{\dot{K}(t)/K(t)}{\varphi(t)}$. When the elasticity of substitution in production is equal to 1, both the savings rate and the consumption to capital ratio remain constant over time.

3. Validation of the facts of Kaldor

According to Kaldor [14] there are six facts that every model of economic growth should explain. Each of them is analysed below. Kaldor says that a model of economic growth must show a continuous growth in the capital-work relationship. This fact is valid in the proposed model since the expression found for $\frac{\partial[K(t)/L(t)]}{\partial t}$ is positive in all cases, thus the capital-work ratio is increasing over time for any value of the elasticity of substitution in production. Another fact that according to Kaldor, every model of economic development must explain is that the product to capital ratio must remain constant over time. The development of the model indicates that this fact is only valid when the elasticity of substitution in production is equal to 1. However, when the elasticity of substitution in production is different from one, under the assumptions on which this is being developed model, economic growth can be given without this fact being explained. When $\sigma_p < 1$, the expression obtained for $\sigma_p < 1$ indicates that the product to capital ratio is increasing over time, even in the long term, a result that agrees with that obtained empirically by Romer [14] who finds that in countries where development path (countries with low income), the product to capital ratio shows a growing trend. On the other hand, when $\sigma_p > 1$, the product to capital ratio shows a decreasing trend over time, which is also at variance with Kaldor's assertion.

Now, the capital-work relationship shows a growing trend over time for any level of α. Therefore, and in view of the fact that the product per worker can be written as $\frac{Y(t)}{L(t)} = \frac{Y(t)}{K(t)}\frac{K(t)}{L(t)}$, in which, when $\sigma_p = 1$, $\frac{Y(t)}{K(t)}$ is constant and $\frac{K(t)}{L(t)}$ is increasing, it can then be assured that when the elasticity of substitution in production is equal to 1, another of the facts of Kaldor, according to which, the per capita product must be increasing over time.

This Kaldor statement is still valid when the elasticity of substitution in production is less than unity, since in that case both $\frac{Y(t)}{K(t)}$ and $\frac{K(t)}{L(t)}$ are both increasing in the time and therefore $\frac{Y(t)}{L(t)}$ turns out to be increasing in time.

Now, when the elasticity of substitution in production is greater than 1, it is observed that $\frac{Y(t)}{K(t)}$ is increasing while $\frac{K(t)}{L(t)}$ is decreasing, which does not allow to describe the behavior of $\frac{Y(t)}{L(t)}$; however, it can be resorted to the fact that when $\sigma_p > 1$, the growth rate of the economy in the long term is given by $G \to \eta + (1 - \sigma_p)\mu$, which is lower than the growth rate of the population and therefore, it can be inferred that the per capita product shows a decreasing trend over time, contrary to Kaldor's claim.

4. Application to developing countries.

The developed model can be extended to be used to explain the economic growth of Colombia and, in general, of developing countries, for this it would be enough to remove the assumption of full employment that has been maintained throughout the work, since that in the developing economies there is not full employment.

If there is no full employment, the salary is fixed exogenously (it is rigid). This exogenously fixed salary determines both the capital-labor ratio and the interest rate. In this case, economic growth will not be determined exogenously by technology and productive force, but endogenously by the expansion of capital, which in turn determines employment, and by the level of savings. Reciprocally, the fixed salary leads to raise the capital-labor ratio and as a result can not employ the entire labor force causing unemployment.

5. Conclusions

When the elasticity of substitution in production is greater than 1, the economy will experience high savings rates that in turn push up the interest rate, which leads to a deterioration in the distribution of income causing the rich to appropriate each time a greater proportion of income than the poor. There is also a decrease in the consumption to capital ratio, that is, over time a decrease in the level of consumption is experienced with respect to the level of capital present in the economy.

Conversely, when the elasticity of substitution in production is less than unity, the saving rates will be increasingly low, which leads to an economy in which growth does not depend on savings and therefore does not worsen the distribution of income. The result obtained here indicates that saving is not a determinant of growth, this is due in turn to the assumption of full employment in the model. If this assumption were removed, saving would become one of the determinants of growth, and in this case, the consumption to capital ratio increases with the passage of time leading to the level of capital present in the economy generates more and more of national consumption.

In a second phase, the analysis of a model with endogenous technological change can be approached and finally the estimation of the model for when the economy is open.

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