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Development of geometric thought

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Abstract. Geometric thinking is a process in which competences are developed to understand three-dimensional space. The experience arises from the need to implement teaching methodologies that allow to optimize both the potential of teachable that has the geometry as the potential of learnability of the student with the purpose of deepening in the learnings so that, by means of the construction of models and the mathematical principle of reconstruction and invention of mathematics, emphasizing spatial thinking and leading the student to recognize geometric objects, geometric spaces and associated transformations to develop mathematical skills while strengthening their geometric thinking.

1. Introduction

The experience arises from the need to implement teaching methodologies that allow to optimize the potential of teachable that has the geometry taking the student to enter the mathematical world, conceptualize and apply the acquired knowledge with the purpose of deepening in the learnings to develop mathematical competences, paying attention to the construction of models and the mathematical principle of reconstruction and invention of mathematics by the student through a process-oriented teaching [1] that favors the development of the five types of mathematical thinking: numerical, metric, spatial, random and variation, but with greater emphasis on spatial thinking leading the student to recognize geometric objects, geometric spaces and associated transformations to develop competences from the strengthening of their geometric thinking.

The proposed methodology involves its applicability in different academic levels with the depth and rigor necessary for the student to develop their geometric thinking based on their mental age and psychological maturity.

The student integrates different knowledge, from the exploration and description of their environment to the application of Euclidean and fractal geometry for the design and estimation of geometric models and application of concepts to the modeling and mathematical sustentation of their work. Likewise, the methodology allows interacting with their peers in the group and with students and professors from other areas, promoting interdisciplinary work, team work and other values in the axiological context that allow the comprehensive training of future professionals.

The methodological proposal uses the theoretical underpinnings of David Ausubel's theory of meaningful learning [2,3], follows basic principles of didactics and multiple intelligences [4,5], and brings the student closer to discovery learning by applying knowledge of specific competences of his professional profile and develops his analytical capacity within the framework of the institutional pedagogical model [6]. In addition, it allows the student to interact in three training contexts: the classroom, the institution and the social environment.

Therefore, the understanding of geometry, its relationship with the environment and nature and the creation of applied geometric models constitute fundamental knowledge to delve into this discipline in order to achieve significant knowledge.

It is also based on competency-based training, understood as the continuous and articulated development of competences throughout life and at all levels of education, it is not enough to have knowledge, but it is essential to go further and use it to solve problems, look for alternatives, produce new knowledge and transform the world in which we live. Thus, to consolidate a quality policy framed in the development of competences implies a fundamental transformation of the pedagogical practices, in the functioning of the Institution and in the role of the actors of the process in such a way that the student is not lost sight of [7].

Competencies are complex processes of performance with suitability in a specific context, that is, actions that follow a specific purpose, where the articulation of different human dimensions is implied for the resolution of problems, fulfilling a series of indicators in a specific context, with Personal and social responsibility [8] also refer to the ability or ability to do tasks or deal with different situations effectively in a given context, for which it is necessary to mobilize attitudes, skills and knowledge at the same time and in an interrelated manner [9].

The development of mathematical competences implies the consolidation of the following general processes: (a) the formulation and resolution of problems, (b) modeling, understood as the detection of schemes that are repeated in everyday, scientific and mathematical situations to reconstruct them mentally (c) communication and its different ways of expressing and communicating problems, conjectures and results that lead to the understanding of phenomena and (d) the independent reasoning of models and materials to bring it to (e) the formulation of propositions and theories, chains argumentative and validations to reach conclusions, applications, verifications and interpretations of models [10].

The development of geometric thought is based on the intuitive notions of spatial geometry focused from the conceptualization of active geometry, this implies transcending the passive geometry of the study of figures and objects, which are apparently static, to the study of a geometry that takes as a basis the "know-how" [11] in a learning methodology that generates academic environments tending to favor the dialogical and critical interaction between individuals [6], allowing the student to reach a significant knowledge.

The proposal is based on the model of Van Hiele's reasoning levels [12], the spatial location of Saiz [13] and the works on the teaching of geometry made by Gallardo and Vergel [1,14].

The incorporation of the work with fractal geometry allows the student to deepen both the explanation and conceptualization of objects in their natural state, such as the construction of geometric models that allow identifying and quantifying their characteristics, as well as the argumentation and support of each one of them the proposed models.

2. Method

The research is descriptive, framed within the quantitative paradigm; It allows describing the development of geometric thinking in students of the master's in mathematics education and provides foundations to each of them to develop proposals to be made with their students at basic, middle and high levels that lead to the development of skills and the strengthening of geometric thinking.

The project is carried out in four phases: a first phase is contemplated by the application of pretest at the beginning of the course, in which the pre-concepts in geometry are valued and a first approach is made to the competences developed by the students, especially in related aspects with spatial knowledge. It presents concepts and methods of Euclidean and fractal geometry, identifying their structural components and establishing their applicability. During this phase, the teacher advises and talks with the students about the subject and clarifies doubts allowing them to discover a logical solution to problematic situations.

The second phase, elaboration and search of didactic material, implies two stages; a first in which the student constructs fractal objects and identifies their properties. In the second, we proceed to the

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search for objects in nature and their description; it is about establishing relationships of these objects with figures and bodies of Euclidean geometry identifying similarities and differences.

The third phase, leads to the approximation by specific methods of Euclidean geometry to the perimeter, surface, area and volume, as appropriate, of the selected objects. The student argues and supports processes used and results obtained.

The fourth phase consists of the use of fractal geometry processes and methods to approximate the geometric properties of the objects being analyzed. The student argues and supports processes used and results obtained.

In all the phases, the student calculates compares and argues, in each case, the topological dimension, Hausdorff dimension, the coating dimension and the fractal dimension. Each student socializes their work and defends it or feeds back with the contributions of classmates and teachers.

3. Results

Students of the master's degree in mathematics education of the "Universidad Francisco de Paula Santander (UFPS)", who are mostly teachers of mathematics in institutions of basic, middle and higher education, were tested for entry behavior in the course of geometry; although the results were satisfactory, it is detected that the student has a good command of flat geometry concepts, but deficiencies in location and understanding of spatial geometry are detected. There is also a high level of understanding and demonstration of properties and theorems related to points, lines and figures of plane geometry.

Information was also collected from students of these teachers and there are conceptual deficiencies in plane and spatial geometry. Thus, this research in mathematics education focused on the field of geometry, accounts for a low level of development of geometric thinking in both teachers and students. In this sense, an intervention process is developed for students of the master's degree course aimed at explaining how the construction of geometric thinking is in the students of their educational institutions; it is expected to improve the level and consolidate learning that involves transition from concrete to abstract thinking in such a way as to encourage the development of geometric thinking.

The work is developed using concepts and procedures of fractal geometry since its study, understanding and application requires the use of the basic and advanced concepts and foundations of Euclidean geometry together with an understanding of the inherent spatial structure. Fractal geometry allows the development of class activities, field practices, the application of geometric methods and, at more advanced levels, the calculation of dimensions of fractal objects and objects of nature that can be approximated by fractal interactions and approaches. to design iterative processes so that through linear algebra applications the linear transformations associated with the construction of fractal objects are established.

In the first instance, we work with the creation of three-dimensional fractals from the folding and cutting of sheets of paper. This activity not only exercises geometric thinking, but also stimulates fine motor skills and allows an approach to new geometric structures. In this activity, students establish relationships between Euclidean geometry and fractal geometry. Identifies elements of self-similarity, iteration and approaches to concepts of Euclidean, topological and fractal attractor and dimension in fractal objects. Students work in groups differentiating between the perimeter dimension and the fractal dimension. When constructing the Koch curve, the perception of a line of infinite length changes because when its attractor is drawn, it approaches a curve of infinite length that can be delimited by a curve of finite length since the Koch curve, also called snowflake, can be inscribed within a circumference or a square.

Students explore nature and identify the need to use fractal geometry methods to solve problems, some of them already known. For example, it is known that the volume of an irregular body is equal to the volume of water displaced; exercise that has been done usually in physics class. However, new questions arise: even when this procedure is valid, is it possible to do it to calculate the volume of a tree? In what container and how is it submerged in the water? Or, how can you calculate the area covered by all the leaves of a tree?

To solve these questions, students identify objects in nature that approximate fractal structures and proceed to approximate perimeters, areas and volumes using methods such as the box dimension and the coating dimension [15] and apply fundamentals of the fractal geometry described by Mandelbrot [16,17].

Students also analyze the related transformations in the plane and space that can be applied to the construction of fractals. These can be worked with a different level of complexity, depending on the academic level. The students elaborate and sustain text in which they describe the procedure used for the construction of a fractal, simulating different academic levels of training. When analyzing Sierpinsky's fractal arrowhead, university students would be able to describe linear transformations that give rise to the fractal and to construct the corresponding equations; if he also has knowledge of computation and programming, he can develop the computer program that from a computer allows drawing the fractal. For students of basic and secondary education, it is possible to access the description of the procedure, that is to say, to describe the changes of scale, rotations and translations necessary to obtain the fractal.

4. Discussion

Fractal geometry allows the development of geometric thinking in students of basic, middle and higher education; For this, it is required that the teacher in the first instance have a good command of specific knowledge in relation to both the theoretical foundations of Euclidean and fractal geometry, as well as their applications; in the second instance it requires that it be creative and that it awakens that creativity in its students when exploring nature in its socio-economic environment. On the other hand, it must incorporate in its educational classroom project, a pedagogical model that stimulates dialogue and criticism [6] so that together with its students achieve intellectual growth around geometry and its applications.

From the description of the fractal structure of an object of nature, the student's creativity is developed together with its capacity for spatial perception and its geometric knowledge; At the same time, it allows us to enter into the formal knowledge of plane and spatial geometry, its postulates, principles, fundamentals and applications.

The work developed encouraged the reading, on the part of the students, on topics related to the time that allowed to identify their relationship with other disciplines; for example, we worked with sampling elements when trying to calculate the area of the leaves of a tree and the volume of the tree from a sample design that would allow, based on a formal sampling process, to select branch and holes and then generalize results to the whole tree. Chemical and ecological knowledge were also incorporated to analyze water and environmental contamination.

In general terms, the development of the academic proposal implies that students observe objects in their natural state, describe them, establish relationships with figures and Euclidean geometric bodies, establish differences and similarities, identify needs for their description, determine fractal methods for their study, apply specific methods of fractal geometry to identify and describe their properties, argue and describe the process carried out, generalize to a mathematical model. All this process is developed in constant dialogue and criticism of the work of their peers with the advice and continuous monitoring of the teacher.

The investigation made it possible to reflect on the pedagogical model and the institutional educational project in which the teacher is immersed, so that he not only deepens in his theoretical foundations, but also applies it and incorporates it in his daily work with his students, thus achieving that they reach a significant learning of geometry and mathematics.

5. Conclusions

The work developed with the teachers in practice, students of the master's degree, allowed to identify different possibilities of applications of geometry both in academic activities and in the exploration of the natural environment in which students find themselves. It is important to highlight that by bringing the student closer to nature, it was possible to identify the latent problems immersed in the study, for example, problems of environmental pollution, deforestation, land use and many more were detected, which allows integrating the study of mathematics with other sciences.

The fractal characterization of nature provides basic information for its study and analysis and the development of academic proposals to jointly develop mathematical thinking, argumentation, and communicative, basic and civic competencies; however, there is still a long way to go in this regard, this will be the subject of an upcoming research project.

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