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Mesoscopic superconductivity in application

J Barba-Ortega¹, J Bautista-Ruiz² and E Sardella³

¹ Departamento de Física, Universidad Nacional de Colombia, Bogotá, Colombia

² Departamento de Física, Universidad Francisco de Paula Santander, San José de Cúcuta, Colombia

³ Departamento de Física, Universidade Estadual Paulista, Baurú, Brazil

E-mail: jjbarbao@unal.edu.co

Abstract. The Ginzburg-Landau theory, applied to superconducting materials is based on thermo-magneto-electro-dynamic concepts as phase transitions that enrich the class on this subject. Thus, in this contribution we expose the Ginzburg-Landau time-dependent equations, show the mathematical form for two nano-scale superconducting systems, one bi-dimensional homogeneous *al* sample with applied external current at zero magnetic field, and one three-dimensional cube in presence of a tilted magnetic field at zero applied current. This analysis shows the applicability of the three and two-dimensional model to superconductors. The conveniently Ginzburg-Landau theory show that the magnetic response behavior of the sample is very useful for applications in fluxtronics, SQUIDS design, magnetic resonance, among others.

1. Introduction

The Ginzburg-Landau theory (GLT) is one convenient tool in studying magnetic response of superconducting. The vortex state and the confinement effects in superconducting samples are very important when its size is of the order the coherence length $\xi(T)$ or to the London penetration depth $\lambda(T)$ (mesoscopic sample). The size of the sample is usually considered to be large enough such that the influence of finite dimensions on their properties are negligible along the external applied magnetic field [1–5]. It was shown that increase of temperature could be measured by nanoSQUIDS devices, Magnetic resonance and levitation, optical traps of vortices in Bose-Einstein condensates, as realized by an arrangement of laser wave. T extra degrees of freedom could open new phenomena in the studied superconducting systems due to different possible optical patterns. GLT published in 1950 satisfactorily described several phenomena related to superconductivity, with a phenomenological and simple aspect, due its the researchers of the area received such work with skepticism, but in 1959, when Gorkov showed that the Ginzburg-Landau equations (GLE) were a particular case of the superconductive theory of BCS first principles, this was popularized in the middle. The interaction of light with superconductivity in most cases leads to local heating of the condensate [6–10]. In 1966, Schmid inserted a temporal dependence on GLE and, thus, the dynamics of systems out of equilibrium could be studied [11–18]. By another handt, there are many experimental and theoretical studies for three-dimensional-systems using the Ginzburg-Landau model has been proven to give a good account of the superconducting properties in samples of several geometries [19–22]. GLT is also used for study the magnetic response of mesoscopic samples in presence of a external dc current, in this systems is possible to visualized the formation of vortex chains by scanning Hall probe



microscopy after freezing the dynamical state through a field cooling procedure at a constant bias current, also, the presence of vortex-anti-vortex ($V - Av$) pair can nucleates in the sample in a non-symmetrical (symmetrical) way due to the assymtry (simetry) of the supercurrent in the sample, so, the $V - Av$ pairs can be controlled experimentally by including defects of other superconducting materials at different critical temperature.

2. Theoretical formalism

In GLT, the superconducting order parameter is a complex function $\psi(\mathbf{r})$ which is interpreted as the pseudo-wave function of the superelectrons $|\psi(\mathbf{r})|^2 = n_s$, where $\psi(\mathbf{r}) = \sqrt{n_s}e^{i\phi}$ is the density of the super-electrons. The Ginzburg-Landau free energy is given by:

$$F = \int \left(\frac{1}{2m^*} \left| \left[-i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right] \psi \right|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{8\pi}B^2 \right) dV = \int \mathcal{F} dV \quad (1)$$

The constants in the equations are: $m^* = 2m$: the mass of a Cooper pair, $e^* = 2e$: the charge of a Cooper pair, (α, β) : phenomenological constants. Variables ψ : the order parameter, \mathbf{A} : the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$, \mathbf{B} : the local magnetic field.

The equations which describe the fundamental properties of the superconducting state described by the complex order parameter ψ , and the vector potential \mathbf{A} can be obtained by using the Euler-Lagrange general equations [19, 20]:

$$\frac{\partial \mathcal{F}}{\partial \bar{\psi}} - \nabla \cdot \left[\frac{\partial \mathcal{F}}{\partial (\nabla \bar{\psi})} \right] = 0 \quad (2)$$

$$\frac{\partial \mathcal{F}}{\partial \mathbf{A}} - \nabla \times \left[\frac{\partial \mathcal{F}}{\partial (\nabla \times \mathbf{A})} \right] = 0 \quad (3)$$

This produces the non time dependent Ginzburg-Landau equations:

$$-\frac{1}{2m^*} \left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right)^2 \psi + \alpha\psi + \beta\psi|\psi|^2 = 0 \quad (4)$$

$$\nabla \times \nabla \times \mathbf{A} = \frac{4\pi}{c} \frac{e^*}{m^*} \text{Re} \left[\bar{\psi} \left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right) \psi \right] \quad (5)$$

The starting point to study the superconductivity of non-equilibrium is the time dependent Ginzburg-Landau equations (TDGL). The TDGL equations are given by:

$$\frac{\hbar}{2m^*D} \left(\frac{\partial}{\partial t} + i\frac{e^*}{\hbar}\Phi \right) \psi = -\frac{1}{2m^*} \left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right)^2 \psi + \alpha\psi + \beta\psi|\psi|^2 \quad (6)$$

$$\frac{4\pi\sigma}{c} \left(\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} + \nabla \Phi \right) = \frac{4\pi}{c} \frac{e^*}{m^*} \text{Re} \left[\bar{\psi} \left(-i\hbar\nabla - \frac{e^*}{c}\mathbf{A} \right) \psi \right] - \nabla \times \nabla \times \mathbf{A}, \quad (7)$$

The constants in the equations are: D : the diffusion coefficient, σ : the electrical conductivity, Φ : the scalar potential. In dimensionless units and in the gauge of zero electric potential $\Phi = 0$, the TDGL equations are given by:

$$\frac{\partial \psi}{\partial t} = -(i\nabla + \mathbf{A})^2 \psi + (1 - T)\psi(|\psi|^2 - 1) \quad (8)$$

$$\frac{\partial \mathbf{A}}{\partial t} = (1 - T) \text{Re}[\psi^*(-i\nabla - \mathbf{A})\psi] - \kappa^2 \nabla \times \nabla \times \mathbf{A} \quad (9)$$

Where ψ is presented in units of $(\alpha/\beta)^{1/2}$, distance in units of coherence length ξ , time in units of $\pi\hbar/96K_B T_c$, \mathbf{A} in units of the upper critical field $H_{c2}\xi$, the Gibbs free energy G in units of $(\alpha T_c)^2/\beta$ and temperature T in units of the critical temperature T_c . κ is the Ginzburg-Landau parameter, Thus, for $\kappa < 1/\sqrt{2}$, the surface energy is positive, and we have the type I superconductors; on the other hand, for $\kappa > 1/\sqrt{2}$, the surface energy is negative, which defines type II superconductors, Is becomes energetically favorable the penetration of quantized flow (vortices) inside. We use the general boundary conditions for the order parameter and magnetic field, they are given by the deGennes boundary conditions $\hat{n} \cdot (-i\nabla + \mathbf{A})\psi|_n = -i\psi/b$, and $\nabla \times \mathbf{A} = \mathbf{B}$ at the surface of the sample. \hat{n} is the normal vector to the surface and b is the deGennes parameter [22–24].

3. Results and discussion

The GLT has been widely used for the study of mesoscopic superconductors. In general, in the steady state the vortices, which in bulk materials are arranged in the network of Abrikosov, in mesoscopic samples, the influence of the surface is so considerable that the arrangement of the vortices follows the symmetry of the system, another exotic behavior is the formation of vortices that have more than one quantum of magnetic flux in their nucleus (giant vortex) [25,26]. We simulated a mesoscopic superconducting prism with lateral transversal section of size $L = 15\xi$, immersed in a magnetic field $H(t)$, the Ginzburg-Landau parameter is $\kappa = \lambda/\xi = 1.3$ (typical value for the Al). The sample presents an bi-dimensional thermal gradient with $T_{left} = T_{down} = 0.25$ and $T_{right} = T_{up} = 0.75$ in the superior view of the long prism.

In Figure 1, the square modulus of the order parameter $|\psi|^2$ and its phase $\Delta\phi$ for $H = 0.88, 1.16$ in the up and downbranch of the magnetic field are depicted. Values of the phase close to zero are given by blue regions and close to 2π by red regions. As is well know, $\Delta T = 0.0$, and the vortices entry always occurs by the central region, we have multi-vortex states with $N = 2, 4(6, 8)$ for $H = 0.88, 1.16$ in the upbranch (downbranch) of H . An increased in H cause the vortex moves towards the interior of the sample.

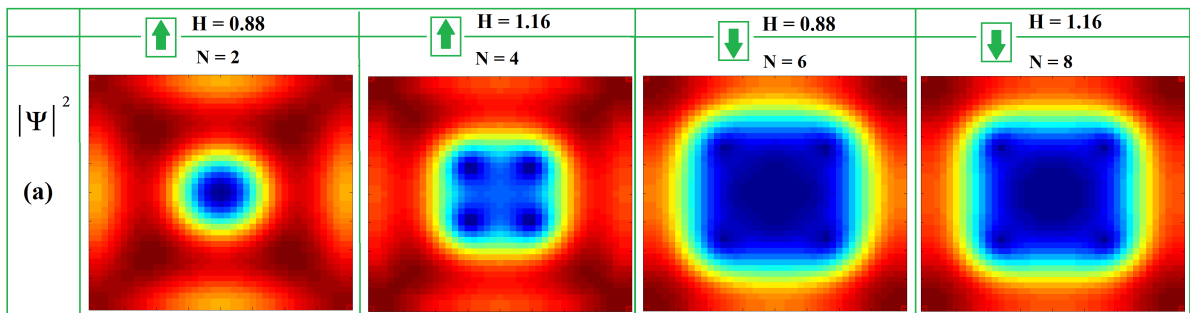


Figure 1. (Color online) Square modulus of the order parameter $|\psi|^2$ at indicated values of H , for $T_{left} = T_{down} = T_{right} = T_{up} = 0.0$ and a $N \rightarrow N + 2$ vortex transition. Blue and yellow regions represent values of the phase $\Delta\phi$ from 0 to 2π and $\psi \sim 0$ to $\psi \sim 1$. Arrows indicates the upbranch and downbranch of the magnetic field.

In Figure 2, the square modulus of the order parameter $|\psi|^2$, its phase $\Delta\phi$ and magnetic induction \mathbf{B} at $H = 0.396, 0.412, 0.456$ are depicted. Values of the phase close to zero are given by blue regions and close to 2π by yellow regions. As we can appreciate the vortices entry always

occur by the region a higher T and multi-vortex states with N to $N + 1$ vortex transition are presented. An increase in the temperature causes the vortex entry at lower magnetic field and then, the vortices tend to move towards the interior of the sample.

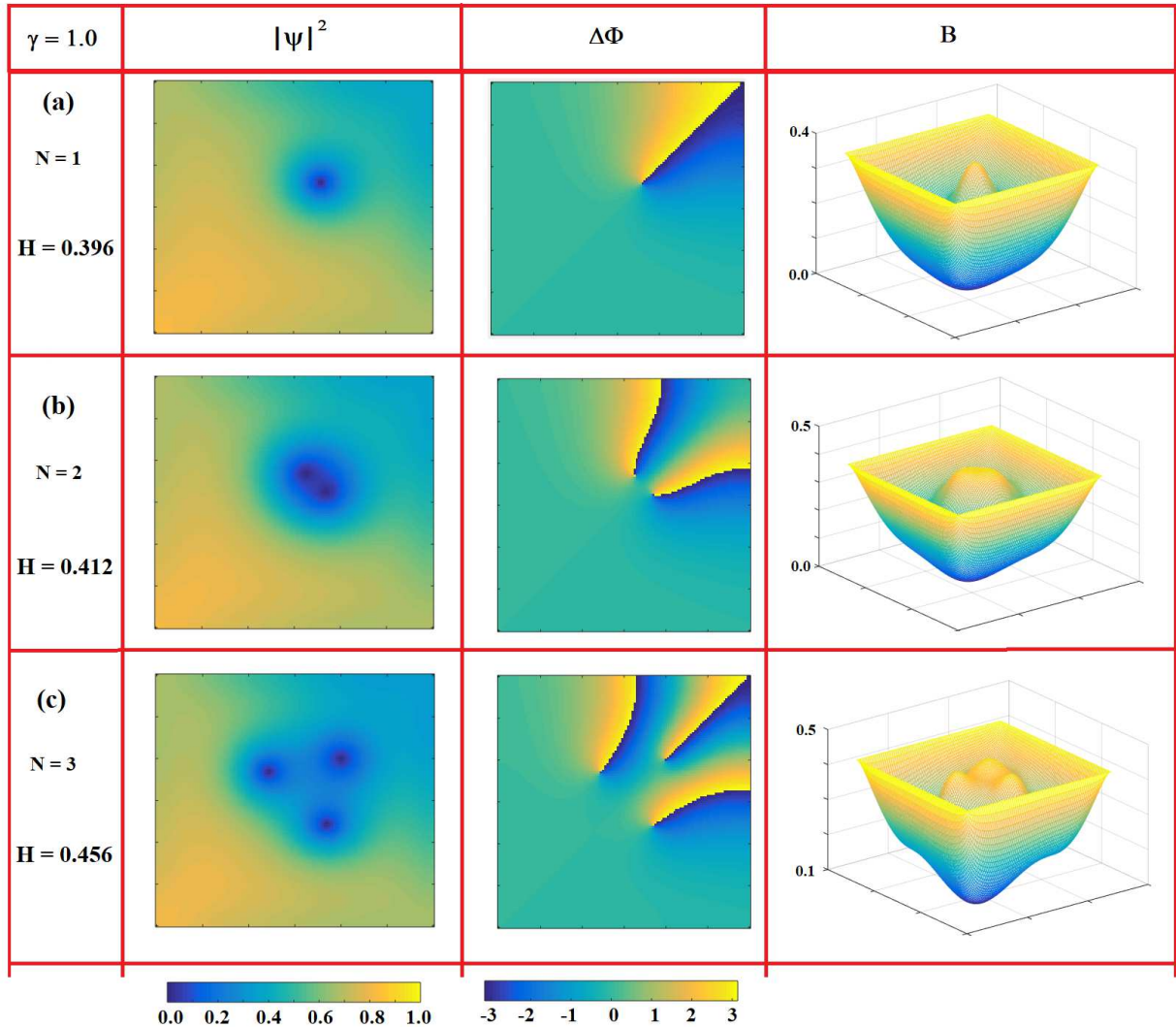


Figure 2. (Color online) Square modulus of the order parameter $|\psi|^2$, its phase $\Delta\phi$ and magnetic induction \mathbf{B} at $H = 0.396, 0.412, 0.456$, $\gamma = 1.0$ indicate a superconductin-normal boundary and $N = 1, 2, 3$ respectively. $T_{left} = T_{down} = 0.25$ and $T_{right} = T_{up} = 0.75$. As is well known, the phase of the order parameter determine the vorticity in a given region, by counting its variation in a closed path around this region. If the vorticity in this region is N , then the phase changes by $2\pi N$.

4. Conclusions

In summary, we solve the Ginzburg-Landau equations for studied the Abrikosov state of a Al superconducting prims. The thermal variation into the sample allows vortices to entry the sample interior by the regions at higher temperature. We also found that the behavior of vortex state present N to $N + 2$ vortex transitions at low magnetic field for a sample submmersed in a homogeneous thermal bath $\Delta T = 0$, and a N to $N + 1$ vortex transition for $\Delta T \neq 0$.

References

- [1] Barba-Ortega J, Sardella E and Aguiar J 2001 *Supercond. Sci. Technol.* **24** 015001
- [2] Barba-Ortega J, Sardella E and Zadorosny R 2018 *Phys. Lett. A.* **382** 215
- [3] Moshchalkov V, Woerdenweber R and Lang W 2010 *Nanoscience and engineering in superconductivity* (New York: Springer)
- [4] Charles Poole Jr 2010 *Handbook of superconductivity* (Amsterdam: Elsevier)
- [5] Barba-Ortega J, Sardella E, and Aguiar J 2015 *Phys. Lett. A.* **379** 732
- [6] León J, Joya M and Barba-Ortega J 2018 *Optik* **172** 311
- [7] Jelić Z, Milošević M, and Silhanek A 2016 *Sci. Rep.* **6** 35687
- [8] Jelić Z, Milošević M, Van de Vodel J , and Silhanek A 2015 *Sci. Rep.* **5** 14604
- [9] Veshchunov I, Magrini W, Mironov S, Godin A, Trebbia J, Buzdin A, Tamarat Ph and Lounis B 2016 *Nat. Commun.* **7** 12801
- [10] Ge J, Gutierrez J, Gladilin V, Devreese J and Moshchalkov V 2015 *Nat. Comm.* **6** 5573
- [11] Landau L and Lifshitz 1980 *E Statistical physics* (Oxford: Pergamon Press)
- [12] Poole C, Farach H, Creswick R 1995 *Superconductivity* (Cambridge: Academic Press)
- [13] Schmidt V 1997 *The Physics of superconductors* (Berlin: Spring-Verlag)
- [14] Schimid A 1966 *Phys. Kondens. Materie* **5** 302
- [15] He A, Xue C, and Yong H 2016 *Supercond. Sci. Technol.* **29** 065014
- [16] Silhanek A, Milošević, Kramer R, Berdiyrov G, Van de Vondel J, Luccas R, Puig T, Peeters F, and Moshchalkov V (2010) *Phys. Rev. Lett.* **104** 017001
- [17] Kramer L and Watts-Tobin R 1978 *Phys. Rev. Lett.* **40** 1041
- [18] Watts-Tobin J, Krähenbühl Y, and Kramer L 1981 *J. Low. Temp. Phys.* **42** 459
- [19] Gropp D, Kaper H, Leaf G, Levine D, Palumbo M, Vinokur V 1966 *J. Comput. Phys.* **123** 54
- [20] Buscaglia G, Bolech C and Lopez A 2000 *Connectivity and superconductivity* ed Berger J and Rubinstein J (Heidelberg: Springer) p 200
- [21] Piacente G, Peeters F 2005 *Phys. Rev. B* **72** 205208
- [22] deGennes P 1994 *Superconductivity of metals and alloys* (New York: Addison-Wesley)
- [23] da Silva R, Milosevic M, Shanenko A, Peeters F and Aguiar J 2015 *Sci. Reports* **5** 12695
- [24] Shanenko A, Aguiar J, Vagov A, Croitoru M and Milosevic M 2015 *Supercond. Sci. Technol.* **28** 054001
- [25] Sardella E, Malvezzi A, Lisboa-Filho P, Ortiz W 2006 *Phys. Rev. B* **74** 014512
- [26] Chibotaru L, Ceulemans A, Bruyndoncx V, Moshchalkov V 2000 *Nature* **408** 833