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Behaviour of the Contact Trace in a Cam-Follower System When the Displacement Law Has Been Designed Using Bézier Curves

Carlos Acevedo Peñaloza[1], Diego Henao León[1] and Piero Rojas Suarez[2]

[1] Mechanical Engineering Program, Engineering Faculty
Universidad Francisco de Paula Santander, Colombia

[2] Civil Engineering Program, Engineering Faculty
Universidad Francisco de Paula Santander, Colombia

Abstract

The study shows the distribution of the contact trace in a cam - follower mechanism in which the Bézier curves of degrees 5, 7 and 9 have been used for the design of displacement of the follower. The equations that describe the movement of the follower and its derivatives are presented, the expressions are formulated for the contact trace, the force exerted on the mechanical assembly are formulated and the results obtained are presented in a model mechanism.

Keywords: contact trace, contact force, displacement law, Bézier's curves

Introduction

In the cam-follower systems, the elements are in permanent contact during their operating cycle. When two bodies are pressed together under the action of a force, a deformation is created on their surface around the point of contact, this region is called the contact surface [10].

In the case of cams, the contact surface changes as the cam makes turns. In the cams, the contact surface is characterized by the contact trace, which represents the width of the deformed zone. The deformation occurs within the plastic limit of the material, which means that the deformity is not permanent. To determine the

size of the contact trace, it is necessary to determine the magnitude of the force generated by each deformation. The contact force depends on the law of displacement of the follower and the pressure angle in each turning section.
In this study we present the Bezier curves used in the design of the law of displacement of the follower with movement of rise and complete descent, in addition, an exemplification of a cam - follower mechanism has been introduced for the application of contact theory in the model.

Methodology

Bézier Curves

The tracer's displacement curves have been designed using curves of Bézier non-parametric, which are base Bernstein and have a Unitary domain [2][3]. These curves have the form

$$B_i^n(u) = \binom{n}{i} u^i (1-u)^{n-1} = C_n^i u^i (1-u)^{n-1}; \quad i = 0, \ldots\ldots, n \tag{1}$$

For this polynomial basis, a function of this type of degree n is expressed as follows

$$\mathrm{b}(u) = \sum_{i=0}^{n} \mathrm{b}_i B_i^n(u) \qquad u \epsilon [0,1] \tag{2}$$

The coefficients b_i define a point on the $b(u)$ curve, which is located equidistant from the abscissa axis at the coordinates

$$b_i = \left(\frac{1}{n}, \mathrm{b}_i\right)$$

and it's known as a checkpoint [2]. The virtues presented by Bézier curves are the intuitive nature and the ease of obtaining continuity in straight stretches; these curves are a good tool in the generation of displacement curves and their derivatives, However, the dependence on the degree of the curve, the difficulty in local control and not being able to guarantee a continuity in the union of curved sections, are some of the limitations of its application [8].

Pressure angle

It is the angle between the axis of the follower rod and the line of action of the force exerted by the cam on the follower. The main recommendation for this angle is that this must be less than 30º, since, starting of this value, the lateral loads on the sliding follower increase considerably [7] [9].
The expression that defines the pressure angle in the cam-follower mechanism is

$$\emptyset = tan^{-1}\left(\frac{s'(\theta)}{s(\theta) + R_o}\right) \tag{3}$$

The maximum value of the pressure angle is found by deriving expression 3 with respect to θ. The values of $s(\theta)\ y\ s'(\theta)$ are the displacement and speed of the follower obtained by replacing the Bézier curves in the displacement law.

Theory of contact

The contact theory has been used to know the expression that determines the contact trace between two cylinders that are considered axially short, that is to say, that their thickness does not exceed the value of the radius of the primary circle, therefore, it is considered that there is flat or two-dimensional stress [1].

Contact force and contact trace

When two curved bodies with elastic characteristics exert pressure on each other by means of a contact force, contact zones capable of supporting the loads generated are developed on them, able to support the generated charges [5]. These areas are in general an elliptical trace that for the case of infinitely rigid cams or cylinders is represented by a rectangle whose area is $2 * \alpha * l$ [1]. The value of the width of the contact trace depends on the properties of the materials and the geometric characteristics of the interacting elements. The expression describing the contact trace is [10].

$$\alpha = \sqrt{\left(\frac{4F_c}{\pi l}\,\frac{\frac{1-\gamma_1^2}{E_1}+\frac{1-\gamma_2^2}{E_2}}{\frac{1}{R_1}+\frac{1}{R_2}}\right)} \tag{4}$$

Donde
F_c: contact force
γ: Poisson ratio of material
E: Modulus of elasticity of the material
l: depth of contact trace
R: radius of curvature of the element

The contact force is given by the expression

$$F_c(t){:}\,\frac{am+\vartheta c+sk+F_{pl}}{cos\ \emptyset} \tag{5}$$

The values of a, ϑ, s are the values corresponding to the acceleration, speed and movement of the tracker respectively. The terms m, k and c are in their order the mass of the tracker including the spring, the constant of the tracker-spring system and the damping constant.

Results and discussion

The displacement law for the cam has been designed with Bézier curves of degrees 5,7,9 and continuity $C^2, C^3, y\ C^4$ respectively and describes a complete upward and downward movement. The functions corresponding to each curve used are shown below for any elevation requirement or cam rotation angle. The figure shows the first section, since the Bézier curves are symmetrical, the downward movement will reflect the upward movement.

Bézier's curved displacement law degree 5. control spots
$b_i = \{0\ 0\ 0\ 1\ 1\ 1\}$

$$b(u) = L[10u^3 - 15u^4 + 6u^5] \tag{6}$$

$$b'(u) = \frac{L}{\beta}[30u^2 - 60u^3 + 30u^4] \tag{7}$$

$$b''(u) = \frac{L}{\beta^2}[60u - 180u^2 + 120u^3] \tag{8}$$

$$b'''(u) = \frac{L}{\beta^3}[60 - 360u + 360u^2] \tag{9}$$

Bézier's curved displacement law degree 7. control spots
$b_i = \{0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\}$

$$b(u) = L[35u^4 - 84u^5 + 70u^6 - 20u^7] \tag{10}$$

$$b'(u) = \frac{L}{\beta}[140u^3 - 420u^4 + 420u^5 - 140u^6] \tag{11}$$

$$b''(\mathrm{u}) = \frac{L}{\beta^2}[420u^2 - 1680u^3 + 2100u^4 - 840u^5] \tag{12}$$

$$b'''(u) = \frac{L}{\beta^3}[840u - 5040u^2 + 8400u^3 - 4200u^4] \tag{13}$$

Bézier's curved displacement law degree 9. control spots
$b_i\{0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\}$

$$b(u) = L[126u^5 - 420u^6 + 540u^7 - 315u^8 + 70u^9] \tag{14}$$

$$b'(u) = \frac{L}{\beta}[630u^4 - 2520u^5 + 3780u^6 - 2520u^7 + 630u^8] \tag{15}$$

$$b''(\mathrm{u}) = \frac{L}{\beta^2}[2520u^3 - 12600u^4 + 22680u^5 - 17640u^6 + 5040u^7] \quad (16)$$

$$b'''(u) = \frac{L}{\beta^3}[7560u^2 - 50400u^3 + 113400u^4 - 105840u^5 + 35280u^6] \quad (17)$$

The factors $L\ and\ \beta$ are respectively elevation and angle of rotation, the term $\mathrm{u} = \frac{\theta}{\beta}$ y b^* are those derived with respect to θ. Figure 1 shows the movement of the follower, the graphs of derivatives 2,3 and 4 represent the speed, acceleration and overacceleration of the follower if the cam rotates at a constant speed.

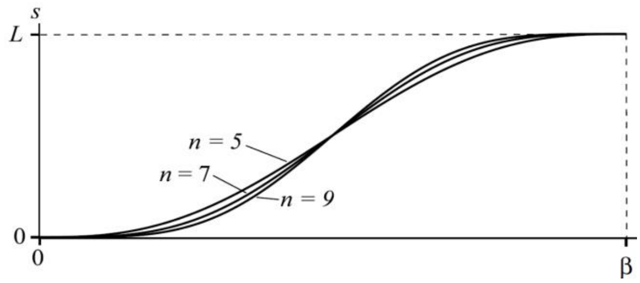


Figure 1. Full upward movement displacement curve

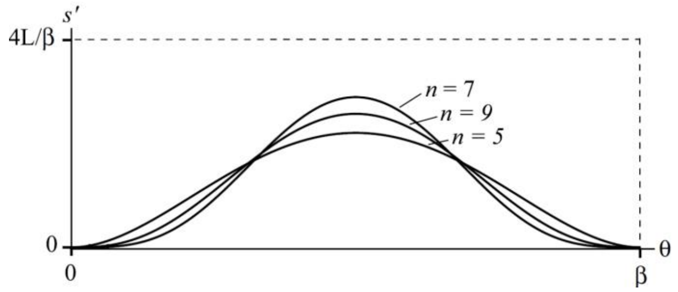


Figure 2. Full upward movement velocity curve

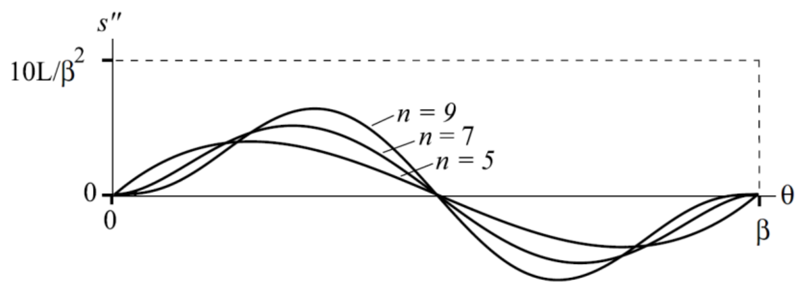


Figure 3. Full upward movement acceleration curve

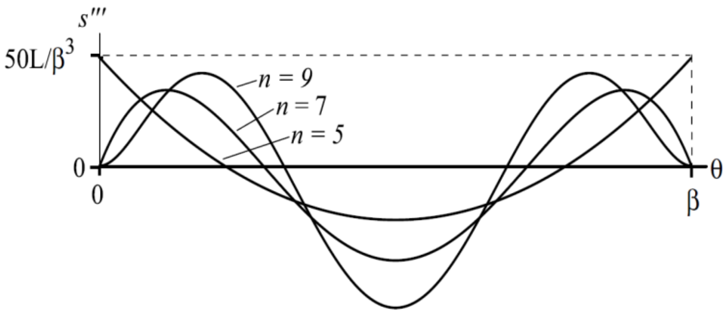


Figure 4. Full upward movement overacceleration curve

As mentioned above, it is important to keep the pressure angle values below 30º to prevent bending of the follower stem [4]. Figure 5 shows the minimum cam angle of rotation value to ensure that the pressure angle recommendation is not exceeded.
The results have been obtained by replacing the Bézier functions corresponding to equation 3 and have subsequently been derived with respect to θ, has been equaled to zero and formulated to determine the ratio $\frac{R_o}{L}$.

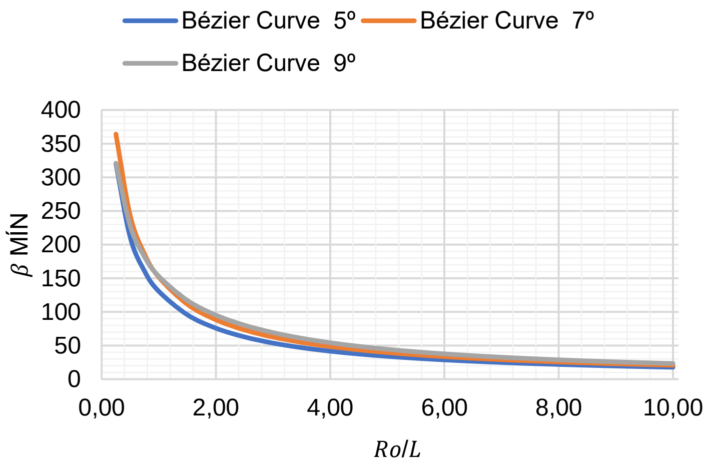


Figure 5. Permissible cam rotation angle not to exceed a pressure angle of 30°.

Analysis model for contact force and contact trace

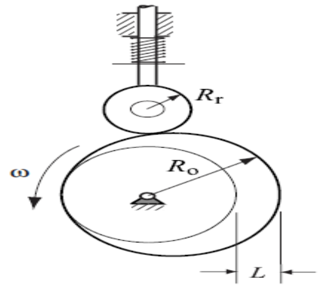
To carry out the analysis, a cam-follower model with force lock has been exemplified, the mass of the follower includes the rod, the roller and the spring, the value of the elasticity factor k and the damping factor c corresponding to 6% of the critical damping is entered [6], and the magnitude of the preload force exerted on the system is also established to guarantee that the follower and the cam do not separate even in negative acceleration sections.

The contact force has been determined by a cinestostatic analysis, it means, by knowing the law of displacement of the tracker, the corresponding values of the force along the movement are determined [7]. For a cam that rotates at constant speed (ω= constant), the contact force takes the form

$$F_c(t): \frac{am\omega^2 + \vartheta c\omega + sk + F_{pl}}{\cos \emptyset} \tag{18}$$

The other conditions of the model studied are [1].

Maximum tracker lift $L = 0{,}01m$
Cam mass $m_l = 0{,}5\ kg$
Follower mass $m_r = 0{,}2\ kg$
Roller radius $R_r = 0{,}01m$
Primary circle radius $R_o = 0{,}02m$
Cam thickness $l = 0{,}01m$
Poisson's ratio of both materials is $\upsilon = 0{,}3$
Equivalent rigidity factor of the system $k = 800\ {}^{N}/_{m}$
System preload $F_{pl} = 1500N$
Damping constant $c = 0{,}06\ c_c$
Angular cam speed $\omega = 20\pi\ {}^{rad}/_{sec}$



Applying the conditions of the model for equation 19 and later for the contact trace gives

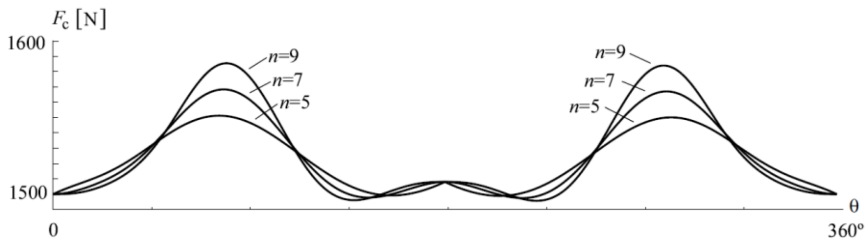


Figure 6. Comparison of the contact force for the Bézier curves using the study model.

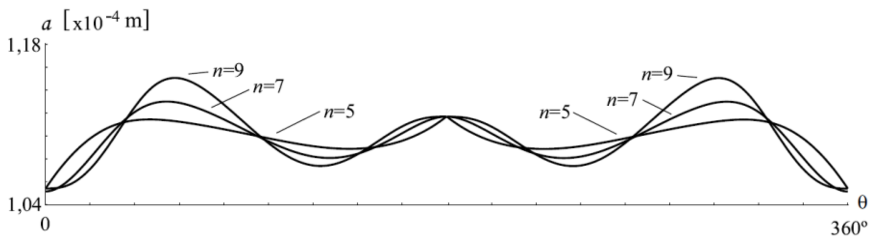


Figure 7. Comparison of the contact trace for the Bézier curves using the study example.

Conclusions

The study of the contact trace and contact force has been developed for a cam-tracking roller mechanism whose displacement law has been carried out through the implementation of Bezier curves of grade 5, 7 and 9.
The contact force and contact trace have been smaller for the cam designed with the Grade 5 Bezier curve compared to its Grade 7 and 9 counterparts.
Grade 5 Bézier curves are the best choice for the travel law design of travel roller trackers because they generate lower loads and less contact stress, which means the assembly will have less risk of fatigue.

References

[1] Acevedo Peñaloza, Carlos Humberto, *Estudio del Ángulo de Presión y Presión de Contacto en Mecanismos de Levas de Seguimiento cuya Ley de Desplazamiento ha sido Diseñada por Bézier Curves*, PhD Tesis, Universidad Politécnica de Cataluña, 2005.

[2] C. Acevedo Peñaloza, E. Zayas, S. Cardona, Introducción al diseño de perfil de levas por curvas de Bézier, *Revista Respuestas*, **9** (2004), no. 1, 33-44.

[3] S. Cardona, D. Clos, *Teoria de Màquines*, (Segunda Edición en catalán). Ediciones UPC. Barcelona. 2000.

[4] David Myszka, *Machines and Mechanisms*, Pearson Educación. México, 2011.

[5] R.C. Juvinall, *Engineering Considations of Stress, Strain an Strength*, McGraw-Hill College, 1967.

[6] M.P. Koster, *Vibrations of Cam Mechanisms.* Phillips Technical Library Seris, Macmillan Press, 1974.

[7] R. Norton, *Diseño de Maquinaria*. Mc Graw-Hill. México, 1995.

[8] G. Reyes Pozo, *Computer Aided Geometric Design Techniques for Cam-Follower Mechanisms*, PhD Tesis, Universidad Politécnica de Cataluña, 2000.

[9] J. E. Shigley, J. J. Uicker Jr., *Theory of Mechanisms and Machines*, Mc Graw-Hill. México, 1988.

[10] S. Timoshencko, J. Goodier, *Theory of Elasticity*, Third Edition,.McGraw-Hill, 1970.

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