Contemporary Engineering Sciences, Vol. 11, 2018, no. 98, 4869 - 4877
HIKARI Ltd, www.m-hikari.com
https://doi.org/10.12988/ces.2018.810536

Determination of the Theoretical Stress Concentration Factor in Structural Flat Plates with Two Holes Subjected to Bending Loads

Carlos Acevedo Peñaloza[1], Jhan Rojas Suarez[2] and Edwin Espinel Blanco[3]

[1] Mechanical Engineering Program, Engineering Faculty
Universidad Francisco de Paula Santander, Colombia

[2] Civil Engineering Program, Engineering Faculty
Universidad Francisco de Paula Santander, Colombia

[3] Mechanical Engineering Program, Engineering Faculty
Universidad Francisco de Paula Santander Ocaña, Colombia

Abstract

Numerous research works have studied the stress distribution in sections with geometric discontinuities and the relationship with the theoretical factor of stress concentration using the finite element method. These works have dealt with topics such as the influence of the length of pieces on the Kt factor in rectangular orthotropic plates; The behavior of the stress concentration factor on short shafts with section changes under bending loads; The theoretical factor of stress concentration in flat parts subjected to tension with "U" grooves; The estimation of the theoretical stress concentration factor Kt for a flat plate with two holes subjected to tension loads. Other studies have compared the stress distribution by applying three methods: elasticity theory, finite element analysis and experimental techniques. Results show that the finite element method is effective in solving the problem.

The goal of this research is to determine the theoretical stress concentration factor for a flat plate with two holes subjected to bending applying the finite element method. The modeling of the plate, the application of the loads, the mesh generation and the simulation of the stress distribution were made using the software

ANSYS® Workbench. Four holes with different sizes were worked and for each of them 10 different values of the distance between centers were taken. In this way the relationship between the factor Kt, the size of the holes and the distance between centers was found. This information is of great importance for the design of mechanical elements that are under these conditions.

Keywords: Bending loads, stress concentration factor, finite element method, structural flat plate, computational simulation

1. Introduction

Due to the geometric discontinuities of all the elements of machines and structural elements of the function for which they were designed, whether they are changes of section, grooves, threads, threads or perforations, it is very important to know the value of the parameter that modifies the distribution of efforts in these discontinuities; this is the theoretical factor of stress concentration. It can be stated that the stress concentration factor Kt is the parameter that indicates how much greater is the real stress in a discontinuity in the geometry of a part compared to the nominal stress. Authors of engineering design texts argue that this parameter is closely related to the geometric characteristics of the parts. Mott [1] asserts that the value of Kt depends on the shape of the discontinuity, the specific geometry of the part and the type of effort. In addition, Budynas and Nisbett [2] explain that the material has no effect on the value of Kt.

There are tables and graphs with the stress concentration factor value for the most common discontinuities. However, this is not the case for flat plates with two bent holes as no studies have been developed to find the Kt value for these specific conditions. However, this research takes as a reference works that have been developed to determine the theoretical stress concentration factor for certain geometries and that have resorted to the analysis by the finite element method since the results are obtained in short times without being necessary to build models or prototypes for experimental practices and without also ignoring the reliability of the results if compared with the experimental methods [3] - [5].

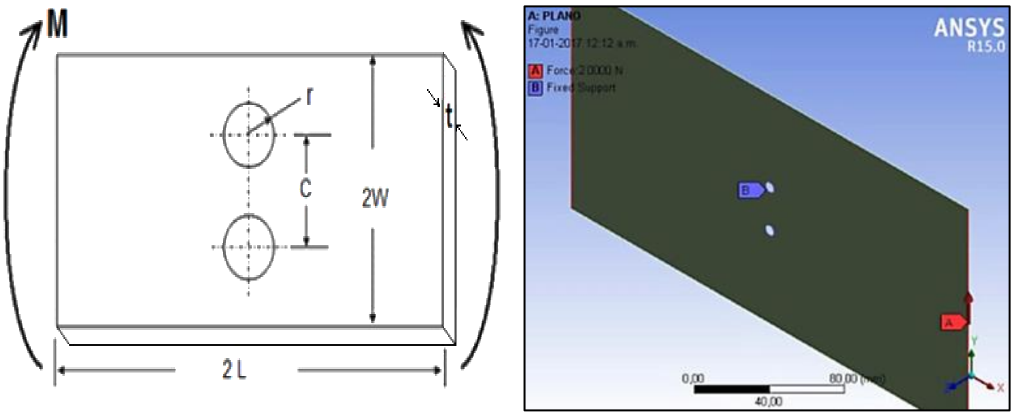
2. Structural Plate Model

The object of study of this research is a rectangular structural flat plate like the one shown in figure 1a. This figure indicates the dimensional parameters of the geometric model; it is defined as follows $2L = 300\ mm, 2W\ = 150\ mm\ y\ t\ =$ 3mm (thickness of the plate).

The diameter of the holes and the distance between their centers were the two variables considered in this study. The analysis was done for four diameter values: $D1\ =\ 12\ mm, D2\ =\ 20\ mm, D3\ =\ 26\ mm\ y\ D4\ =\ 40\ mm.$

The dimensions of the plate have been taken into account taking into account that the effects of maximum loads and stresses had to occur in the vicinity of the holes.

In addition, consideration was given to the fact that the stresses generated were in a range that did not exceed the elastic limit of the material. The study was performed for a carbon steel isotropic plate.
The plate is loaded with two equal forces, one at each end to generate a maximum bending moment in the middle of the mechanical element. The analysis was carried out with a load of 20kN, which generates a moment of 3 kN-m on the Z-axis of the coordinate system established in the model as shown in figure 1b below.



a) . b)

Figure 1. a) Geometric model of flat plate with two holes. b) Model with loads

The ANSYS® Workbench software has been used to model the geometry, generate the mesh of the plate and obtain the simulation of the forces. In this case, it was started with the option "generate mesh" to create the mesh automatically. Subsequently, the mesh is refined to obtain, on average, elements between 2.5 and 0.3 mm.
Figure 2 shows the combination of size and orientation of the quadrilateral elements. You can see how a greater number of smaller elements are generated in the vicinity of the holes.

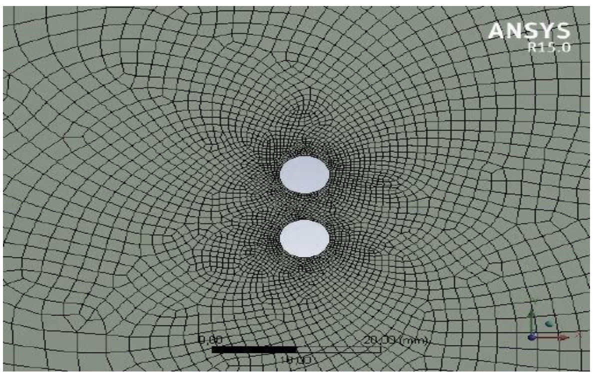


Figure 2. Generated mesh for holes with a diameter of 12 mm.

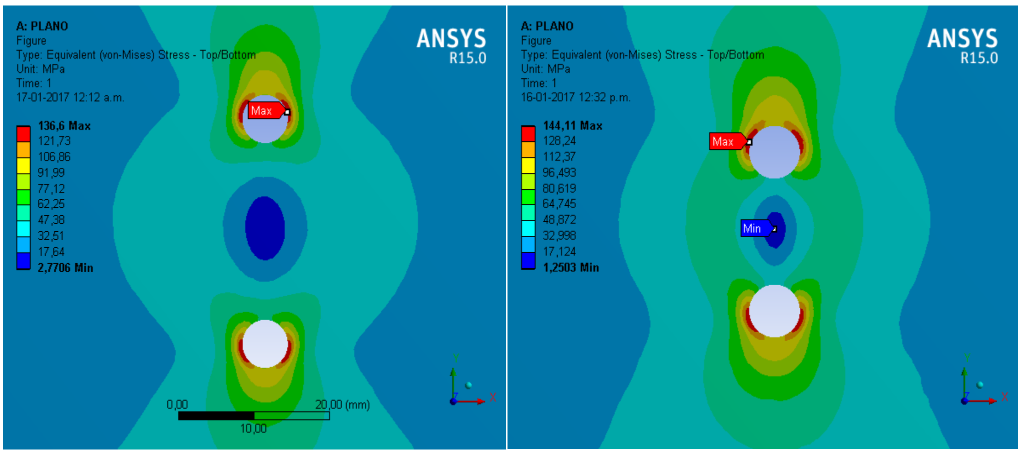
3. Computational Simulation of Stresses

Previously stress concentrators were determined by means of experimental techniques, but recently space has been given to numerical simulations, which have gained great acceptance for the results obtained. Avilés [6] states that stress concentration factor tables have traditionally been obtained experimentally through techniques such as photoelasticity and more recently through finite elements with excellent results. It is possible to say that the analysis by numerical simulations is used with very good precision for the study of stresses in elastic bodies; this method calculates the total potential energy by adding the energy of internal deformation and the potential energy of external forces. This is a great advantage compared to the traditional method of the elasticity theory that in many occasions requires the solution of a large number of differential equations with a high degree of complexity according to the conditions of the geometry being analyzed.

In the present work the geometric model of the plate with the properties of the material was defined, the points of support of the model were fixed and the loads were applied. In this case, "fixed support" type supports were placed, and the model was loaded as described above.

Figures 3a and 3b show the analysis report. The module of the represented efforts is delivered. Ten different values of the distance between centers were taken and, in all cases, the maximum efforts were presented in the same region of the contour of the orifices, expected aspect because it is in this area of the plate where is the geometric discontinuity that gives concentration of efforts. In the same way, the minimum effort was given for all cases, in the central plane of the plate.

This is also consistent with the theory of flexion since it is in this plane where the neutral axis of the part is located; these results give validity to the computational numerical simulation.



a) b)

Figure 3. a) Stress analysis report for D = 12 mm and C = 28 mm. b) Stress analysis report for D = 12 mm and C = 18 mm.

Figure 4 corresponds to a representation of the behavior of the plate in relation to the deformation. By observing the way in which the stresses are distributed over the whole area of the plate and, if compared with the scale bar indicating the maximum and minimum values, it can be inferred that the degree of deformation is minimum for the applied load.
The calculation of the theoretical stress concentration factor Kt was made by means of equation (1), the relationship between maximum stress and nominal stress. As Clavijo and Montoya [7] did, the maximum stress was taken from each of the simulated cases, while the nominal stress was calculated for the pure bending model with equation (2).

$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \tag{1}$$

$$\sigma_{nom} = \frac{My}{I} \tag{2}$$

Where K_t is the stress concentration factor, σ_{max} is the real maximum stress in the discontinuity and σ_{nom} is the nominal stress in the section of the mechanical part. In addition, M is the bending moment; and it is the distance between the neutral axis of the plate and the point of calculation of the stress; and finally, I corresponds to the inertia of the transversal section, for this case it is of 8,4375E-7 m^4.

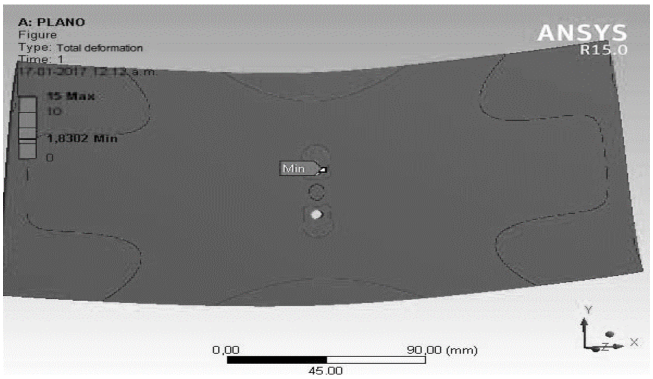


Figure 4. Strain analysis for the plate.

Figures 5 and 6 show the characteristics of the holes, the different values of the distance between centers, nominal stresses, minimum stresses and their respective value of the Kt.

D1 = 12 mm			r = 6 mm		D2 = 20 mm			r = 10 mm	
C (mm)	C/r	σ_{nom} (MPa)	$\sigma_{máx}$(MPa)	K_t	C (mm)	C/r	σ_{nom} (MPa)	$\sigma_{máx}$(MPa)	K_t
10	1.67	14.22	159.38	11.21	20	2	53.33	201.61	3.78
12	2.00	16	154.16	9.64	22	2.2	56.89	200.32	3.52
14	2.33	17.78	149.74	8.42	24	2.4	60.44	197.11	3.26
16	2.67	19.56	147.02	7.52	26	2.6	64	195.81	3.06
18	3.00	21.33	144.11	6.76	28	2.8	67.56	193.3	2.86
20	3.33	23.11	144	6.23	30	3	71.11	191.12	2.69
22	3.67	24.89	140.3	5.64	32	3.2	74.67	189.65	2.54
24	4.00	26.67	139.53	5.23	34	3.4	78.22	188.58	2.41
26	4.33	28.44	137.18	4.82	36	3.6	81.78	192.18	2.35
28	4.67	30.22	136.6	4.52	38	3.8	85.33	191.83	2.25

Figure 5. Stress concentration factor for D = 12 mm and D=20.

The stress concentration factors for all the holes under study are presented in Figure 7 where the variation with respect to the relationship between the distance between centers and the radius of the hole is shown.

D3 = 26mm			r = 13 mm		D4 = 40mm			r = 20 mm	
C (mm)	C/r	σnom (MPa)	σmáx(MPa)	K_t	C (mm)	C/r	σnom (MPa)	σmáx(MPa)	K_t
28	2.15	72.89	165.93	2.28	30	1.5	88.89	148.46	1.67
30	2.31	76.44	164.48	2.15	32	1.6	92.44	148	1.6
32	2.46	80	163.59	2.04	34	1.7	96	147.57	1.54
34	2.62	83.56	163.41	1.96	36	1.8	99.56	147.17	1.48
36	2.77	87.11	162.23	1.86	38	1.9	103.11	146.77	1.42
38	2.92	90.67	161.88	1.79	40	2	106.67	146.4	1.37
40	3.08	94.22	161.18	1.71	42	2.1	110.22	146.05	1.33
42	3.23	97.78	160.83	1.64	44	2.2	113.78	145.72	1.28
44	3.38	101.33	160.52	1.58	46	2.3	117.33	145.41	1.24
46	3.54	104.89	160.1	1.53	48	2.4	120.89	145.14	1.2

Figure 6. Stress concentration factor for D = 36 mm and D = 40 mm.

In all the cases studied, it was found that the stress concentration factor decreases with increasing hole spacing. However, it should be noted that although the change in centre distance was made for all cases with increments of 2 mm, the decrease in Kt does not occur in the same proportion.

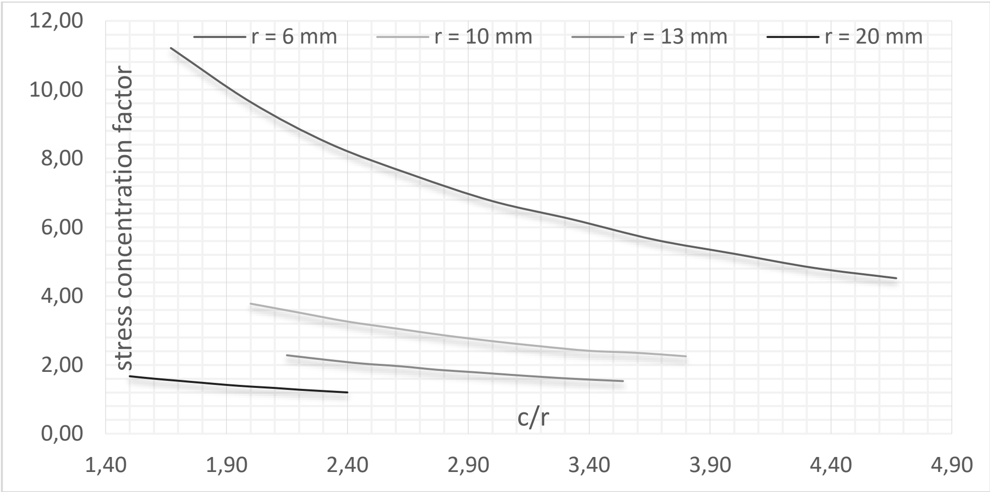


Figure 7. Curve of stress concentration factor for a flat plate with four-hole sizes

4. Discussion of Results

As it has been mentioned in all the cases studied, it has been found that the stress concentration factor decreases with increasing hole spacing. For D1 the decrease of Kt was 60%, for D2 it was 40%, for D3 it was 32% and for D4 the reduction was 28%. It can be seen how much the Kt is bound as a function of C in the case of smaller diameter holes. This can be seen in figure 7 where the curve corresponding to the smaller size of holes shows a greater slope.
The maximum value of the stress concentration factor was given for the smallest diameter of holes with the smallest distance between centers (Kt = 11.21) and the lowest value of the Kt was given for the largest size of holes with the largest distance between centers (Kt = 1.2). A marked tendency of the stress concentration factor towards a single value has not been found; but the decrease of stress intensification each time the two holes are separated became evident.
For the larger holes, the variation in the value of the maximum stresses in relation to the change in the distance between centers is not very marked, as opposed to the holes with smaller diameters, where a significant variation could be observed.
The numerical simulations gave expected results if compared with the theoretical bases, in this case, in relation to the theory of stresses under bending loads. As indicated in the engineering design texts [8], [9], on the central plane of the plate, the magnitude of the stresses was minimal since the neutral axis of the geometry is located on that plane. In addition, the location of the maximum stresses on the contour of the holes also gives validity to the numerical simulation because it is in this region of the plate where the stress concentration generating discontinuity is located.

5. Conclusions

For flat structural plates with two holes subjected to bending loads, the stress concentration factor is related to the distance between the holes. That is, if the spacing of the holes is increased, the Kt value decreases and if the distance between the holes is reduced, the Kt value increases.
For structural flat plates with two holes subjected to bending loads, regardless of the size of the holes, the maximum stresses have a higher value when the distance between the holes becomes smaller.
In structural flat plates with two holes subjected to bending loads, when the holes are large, there is very little change in the value of the maximum stresses if the holes are moved away or closer; whereas for small holes, there is a noticeable change in the value of the maximum stresses as the distance between the holes varies.
For structural flat plates with two holes subjected to bending loads, the theoretical stress concentration factor values are higher in smaller holes. Therefore, the larger the holes, the lower the value of Kt.

References

[1] R. L. Mott, *Diseño de Elementos de Máquinas*, 4ª ed,, México D. F.: Pearson educación, 2006.

[2] R. G. Budynas, J. K. Nisbett, *Diseño en Ingeniería Mecánica de Edward Shigley*, 9ª ed., México D.F.: McGraw-Hill, 2012.

[3] F. Roldan, U. Bastidas, Estudio experimental y por elementos finitos del factor de concentración de esfuerzos producido por un agujero en una placa plana, *Dyna*, **69** (2002), no. 137, 1-8.

[4] M. Sánchez, N. Troyani, Factor teórico de concentración de esfuerzos en piezas cortas de materiales anisotrópicos, *XIII Congreso Sobre Métodos Numéricos y sus Aplicaciones*, Bahía Blanca, Argentina, (2003).

[5] J. Jaya, I. Meza, Estudio de la deformación en placas largas con dos agujeros, bajo la acción de fuerzas axiales, para determinar los concentradores de esfuerzos Kt. Trabajo de grado ingeniería mecánica. San José de Cúcuta: Universidad Francisco de Paula Santander, 2014.

[6] R. Avilés, *Análisis de Fatiga en Máquinas*, Madrid: Thomson Editores, 2005.

[7] J. Clavijo, C. Montoya, *Cálculo del Factor de Concentración de Esfuerzos en Placas Planas Variando las Ecuaciones de Forma por Medio del Método de Elementos Finitos*, *Trabajo de Grado Ingeniería Mecánica*, Pereira: Universidad Tecnológica de Pereira, 2015.

[8] A. Deutschman, W. Michels, C. Wilson, *Diseño de Máquinas - Teoría y Práctica*, 2ª ed., México D.F.: Compañía editorial continental S.A., 1978.

[9] V. M. Faires, *Diseño de Elementos de Máquinas*, Barcelona: Montaner y Simon S.A.

Received: October 18, 2018; Published: November 22, 2018