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Principal Stresses on the Contact Surface of a Cam-Tracker Mechanism Designed Using Bézier Curves

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Abstract

The study exposes the variation of the main stresses in a cam mechanism with roller follower whose profile has been designed by means of Bézier curves. The contact theory for the cam case is proposed and the distribution of the stresses throughout the cam turning cycle is presented, applied to an established study model.

Keywords: principal stresses, contact theory, Bézier curves, displacement law

Introduction

The cam-follower mechanisms are widely used in engineering because, when compared to four-bar mechanisms, they occupy less space and can develop different types of movements. [5][8]. The displacement that defines the movement of the cam follower is commonly designed using algebraic and trigonometric polynomials. The academic literature used to develop cam design uses harmonic and cycloidal curves, which belong to this type of functions [2][4].

Bezier curves have been used to improve the performance of elements used in engineering, among its applications is the design of structural surfaces [3], optimization of extrusion dies [12] and sound amplification of horns [6], However,

little has been deepened in the use of the Bézier curves for the design of the law of displacement of cam trackers.
The article presents the distribution of the main stresses on the surface of a cam whose law of displacement has been designed by means of Bézier curves, exposes the functions used in the process of designing the law of displacement for a complete upward and downward movement, presents the theory of contact applied to cams and then raises the conditions of the mechanism for the application of this theory.

Methodology

Displacement Law and Bézier Curves

The displacement law defines the position of the follower during the cam rotation cycle, depending on the elevation requirements of the follower and the functions used for the construction of its displacement curve, the geometric shape of the cam is determined. For the law of displacement Bézier curves of grades 5, 7 and 9 have been used, these types of curves are classified as Bernstein base polynomials and have the form [2]

$$B_i^n(u) = \binom{n}{i} u^i (1-u)^{n-1} = C_n^i u^i (1-u)^{n-1}; \quad i = 0, \ldots\ldots, n \qquad (1)$$

The Bézier curves have been recommended to develop curves of displacement, speed and acceleration. [8].

Theory of contact applied to cams

A disc cam mechanism with roller follower is studied as a pair of parallel rollers in rotation movement. When two cylinders are in contact and experience a rotational movement, the force that holds them together generates a deflection in the area where the contact between the two elements occurs [7]. In general, this phenomenon occurs within the plastic zone of the material, therefore, the affected area recovers its initial condition at the end of the contact. If this happens throughout the cylinder rotation and is repeated as many times as the cylinder rotation, the elements will experience a series of stresses called contact stresses or Hertz stresses.
In the case of a cam-follower mechanism, two conditions are considered: flat stress and flat deformation stress, where the first corresponds to axially short cylinders and the second to axially long cylinders. The system related in the present work corresponds to the flat stress condition. [1].

Rolling contact forces with tangential force

Pure rolling contact forces with tangential load appear when the mechanism presents a rotational movement and sometimes a sliding movement. This situation is due to variations in the speed of the mechanism and possible problems in starting it.

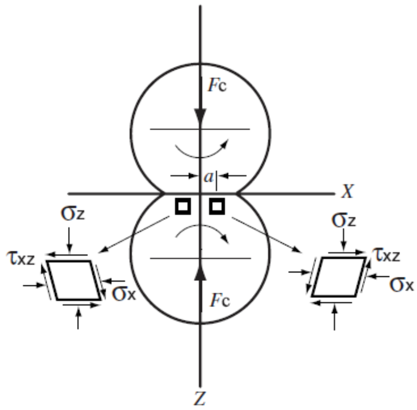

Figure 1. Contact Forces on Combined Load Rolls

The resulting stresses are obtained by adding the stresses produced by the normal load and the tangential load in their respective directions [11], for the case under study, the stresses are presented only in two directions as shown in Figure 1. The total stresses are expressed as follows

$$\sigma_x = \sigma_{xn} + \sigma_{xt} \tag{2}$$

$$\sigma_z = \sigma_{zn} + \sigma_{zt} \tag{3}$$

$$\tau_{xz} = \tau_{xzn} + \tau_{xzt} \tag{4}$$

Stresses on the contact Surface

The forces present on the surface of the contact zone are expressed as [8][10]

- For $|x| \leq a$

$$\sigma_{xn} = -\mathrm{p}_{max}\sqrt{1-\frac{x^2}{a^2}} \quad else \quad \sigma_{xn} = 0 \quad \sigma_{zn} = \sigma_{xn} \quad \tau_{xz} = 0 \tag{5}$$

$$\sigma_{xt} = -2f_{max}\frac{x}{a} \qquad \sigma_{zt} = 0 \tag{6a}$$

$$\tau_{xzt} = -f_{max}\sqrt{1-\frac{x^2}{a^2}} \qquad \tau_{xzt} = 0 \tag{6b}$$

- For $x > a$

$$\sigma_{xt} = -2f_{max}\left(\frac{x}{a} - \sqrt{\frac{x^2}{a^2} - 1}\right) \quad (6c)$$

- For $x < -a$

$$\sigma_{xt} = -2f_{max}\left(\frac{x}{a} + \sqrt{\frac{x^2}{a^2} - 1}\right) \quad (7)$$

The terms f_{max}, a y p_{max} are respectively the unit tangential force, the width of the contact zone and the maximum pressure exerted on the friction area. Their values are calculated using the expressions

$$f_{max} = \mu \mathrm{p}_{max} \quad (8)$$

$$\mathrm{p}_{max} = \frac{2F_c}{\pi a l} \quad (9)$$

$$\alpha = \sqrt{\left(\frac{4F_c}{\pi l}\,\frac{\frac{1-\gamma_1^2}{E_1} + \frac{1-\gamma_2^2}{E_2}}{\frac{1}{R_1} + \frac{1}{R_2}}\right)} \quad (10)$$

As can be seen, equations 9 and 10 have in common the factor F_c, which represents the magnitude of the contact force exerted on the mechanism. The contact force depends on the characteristics of the movement of the tracker and the pressure angle of the assembly.
The equation used to determine the magnitude of the force applied to the mechanism throughout the cycle is given by

$$F_c(t): \frac{am\omega^2 + \vartheta c\omega + sk + F_{pl}}{cos\ \emptyset} \quad (11)$$

The terms a, ϑ, s correspond to the acceleration, speed and elevation of the tracker respectively. The letters m, k and c represent respectively the mass of the tracker system, the spring constant and the damping constant.

Study model
The Bezier curves have been formulated for any requirement of elevation of the tracker and angle of rotation of the cam, therefore it is necessary to establish the geometric and operating characteristics of a mechanism cam - tracker and thus apply

the theory of contact to determine the stresses on the surface that experiences the assembly during a cycle of operation. The parameters describing the model mechanism used in this study are shown in Table 1

Características del modelo	**value**
Maximum tracker lift $\boldsymbol{L}$	$0{,}01m$
Cam mass $\boldsymbol{m_l}$	$0{,}5\ kg$
Tracker mass $\boldsymbol{m_r}$	$0{,}2\ kg$
Roller radius $\boldsymbol{R_r}$	$0{,}01m$
Primary circle radius $\boldsymbol{R_o}$	$0{,}02m$
Cam thickness $\boldsymbol{l}$	$0{,}01m$
Poisson's ratio $\boldsymbol{\upsilon}$	$0{,}3$
Equivalent rigidity factor $\boldsymbol{k}$	$800\ {}^{N}/{}_{m}$
System preload $\boldsymbol{F_{pl}}$	$1500N$
Damping constant $\boldsymbol{c}$	$0{,}06\ c_c$
Angular cam speed $\boldsymbol{\omega}$	$20\pi\ {}^{rad}/{}_{seg}$

Table 1. Characteristics of the model mechanism

Results and discussion

The curve that describes the position of the tracker has been designed using Bézier curves, for this purpose have been used curves of grade 5, 7 and 9. The movement of the tracker consists of two complete movements, ascent and descent, the movements present symmetry, which means that the second is a reflection of the first. Figure 2 shows the upward movement of the tracker for the different degrees of Bézier curve used.
As it has been mentioned before, the curves have been formulated to satisfy any requirement of elevation or rotational angle of the cam. The functions proposed to describe the law of displacement of the follower are presented below

Law of curved displacement of Bézier grade 5.

$$b(u) = L[10u^3 - 15u^4 + 6u^5] \tag{12}$$

Law of curved displacement of Bézier grade 7.

$$b(u) = L[35u^4 - 84u^5 + 70u^6 - 20u^7] \tag{13}$$

Law of curved displacement of Bézier grade 9.

$$b(u) = L[126u^5 - 420u^6 + 540u^7 - 315u^8 + 70u^9] \tag{14}$$

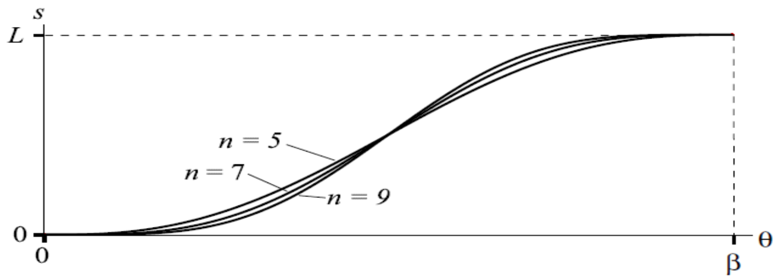

Figure 2. Full upward movement displacement curve

To calculate the main stresses in the mechanism, it is necessary to derive equations 12, 13 and 14 to find expressions corresponding to the velocity, acceleration and over-acceleration that the mechanism experiences during a work cycle. By deriving with respect to u and considering that $\mathrm{u} = \frac{\theta}{\beta}$, it has been obtained

Derived from the law of curved displacement of Bézier grade 5

$$b'(u) = \frac{L}{\beta}[30u^2 - 60u^3 + 30u^4] \tag{15}$$

$$b''(u) = \frac{L}{\beta^2}[60u - 180u^2 + 120u^3] \tag{16}$$

$$b'''(u) = \frac{L}{\beta^3}[60 - 360u + 360u^2] \tag{17}$$

Derived from the law of curved displacement of Bézier grade 7

$$b'(u) = \frac{L}{\beta}[140u^3 - 420u^4 + 420u^5 - 140u^6] \tag{18}$$

$$b''(\mathrm{u}) = \frac{L}{\beta^2}[420u^2 - 1680u^3 + 2100u^4 - 840u^5] \tag{19}$$

$$b'''(u) = \frac{L}{\beta^3}[840u - 5040u^2 + 8400u^3 - 4200u^4] \tag{20}$$

Derived from the law of curved displacement of Bézier grade 9

$$b'(u) = \frac{L}{\beta}[630u^4 - 2520u^5 + 3780u^6 - 2520u^7 + 630u^8] \tag{21}$$

$$b''(\mathrm{u}) = \frac{L}{\beta^2}[2520u^3 - 12600u^4 + 22680u^5 - 17640u^6 + 5040u^7] \quad (22)$$

$$b'''(u) = \frac{L}{\beta^3}[7560u^2 - 50400u^3 + 113400u^4 - 105840u^5 + 35280u^6] \quad (23)$$

the functions have been developed with the elevation value provided by table 1. As an initial step the value of the pressure angle has been determined using equation 18 to later determine the values of the contact force and its dependent equations. The values of β have been selected considering the recommendation not to exceed the angle of pressure to more than 30º [1].

$$\emptyset = tan^{-1}\left(\frac{s'(\theta)}{s(\theta) + R_o}\right) \quad (24)$$

Developing the system of equations has obtained the variation of the stresses in the mechanism

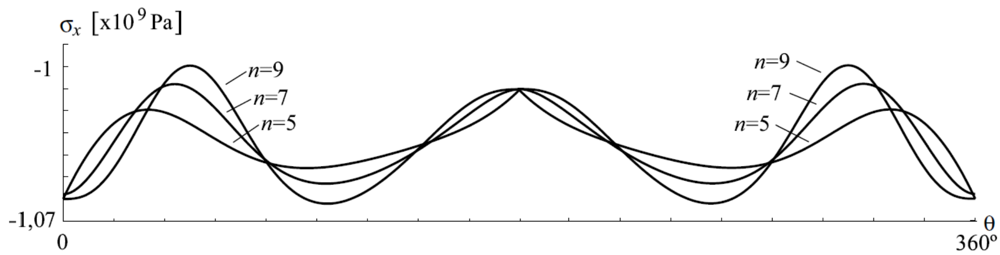

Figure 3. Distribution of surface stress in x-direction

The combined load condition makes the stresses in the x-direction greater, the negative value of the stresses in the x-direction makes it possible to deduce that the stresses produced by the normal load predominate over the sliding stresses. [1].

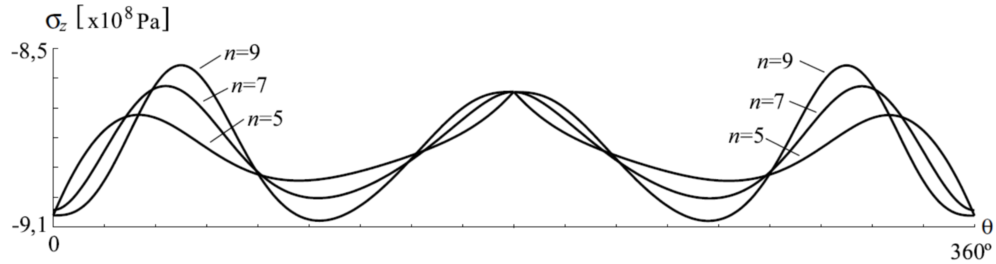

Figure 4. Distribution of surface stress in x-direction

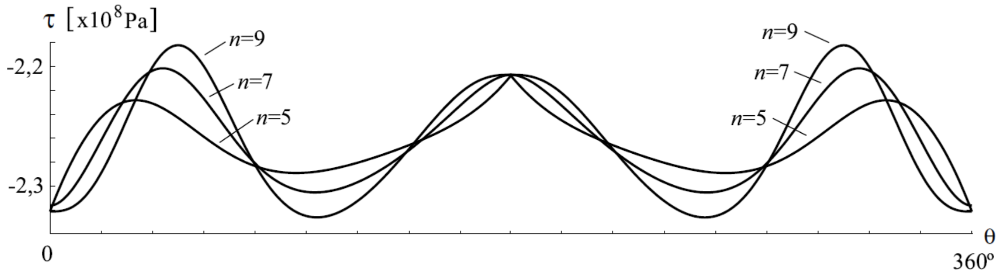

Figure 5. Distribution of shear force on the surface

Although the values of the stresses differ, similarity is observed in the variation of these, this situation is since both present a direct relationship with the contact force and an inverse relationship with the contact trace [1].

Conclusiones

The variation of the main efforts in a cam-follower mechanism has been presented, whose displacement law has been designed by means of Bézier curves of degrees 5, 7 and 9.

From the results obtained, it has been determined that the grade 5 curves present better behavior in comparison with their grade 7 and 9 counterparts since the magnitude of the stresses is smaller, which generates better conditions in the final design of the mechanism.

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