Understanding of derivative as an essential part of the study of differential calculus

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Understanding of derivative as an essential part of the study of differential calculus

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Abstract. The research aimed to design a teaching strategy and analyze benefits of its implementation in the use of technologies like GeoGebra and interactive materials that offers the web as enabling tools and teaching as mediation is aimed at students in grade 11 middle education. Tests pre-test and post-test and two experimental groups were applied control. It was verified that the teaching strategy has a positive influence on learning the experimental group improved the level of understanding of the concept of derivative and thus academic performance, in contrast to the control group to which is applied the traditional methodology.

1. Introduction

Research on teaching and learning calculus and development of variation thinking students have allowed to demonstrate the difficulties and problems at the time to understand their concepts. The fact of working the concept derived from the concept of limit represents serious difficulties, although some students understand and acquire a domain to derive and apply the concepts in solving problems, not all acquire these skills and even outstanding students have difficulty explaining the concepts of limit, the derivative, which makes it clear misconceptions [1]. The main difficulties encountered in understanding the fundamental concepts of calculus [2], they are related to the conceptualization of the notion of limit; teaching and related conflicts regarding the domain of algebraic thinking.

According to the above, the multiple detected shortcomings demonstrate the reality of the educational context in which ways to address the processes of understanding of fundamental concepts, taking as its starting point the notion that the student has the concept to delve into the development of a structure that strengthens the cognitive and conceptual development. The derivative is a fundamental concept of differential calculus; this can be interpreted geometrically as the slope of the tangent to the curve at a given point and physically as a rate of change. Explanations about this phenomenon, framed by the fact that the learning strategies students adapt to the acquisition of skills and problem-solving skills successfully in different contexts. In this regard, it is necessary to design and implement strategies that allow students to have a solid conceptual framework, by developing procedures and activities carried out intentionally inside and outside the classroom, using technological tools favoring the teaching, learning.

Acceptance that today has the mathematical knowledge in the social context implies a challenge for teaching mathematics to the fact sharpen your strategies to promote the development of mathematical thinking in students. In this vein, we can interpret the development of mathematical thinking, as a

spontaneous reflection that mathematicians made about the nature of knowledge and the nature of the process of discovery and invention in the area. Finally, in the process of teaching and learning concept derived it is essential to consider that reaching understanding the student must intuit, be creative, master preconception and the concept, recognize and interpret the concept image, demonstrate or prove, detect difficulties.

2. Methodology

To investigate the effect caused by the teaching strategy based on the use of new technologies to improve academic performance and other factors that affect learning and understanding the concept of derivative aimed at middle school students of "Colegio José María Córdoba" in Durania, Norte de Santander, Colombia, and the proposal is part of an approach quantitative with qualitative approach support. The research, supported quantitative approach with qualitative, quasi-experimental design. The phases contemplate analysis conceptions: A first phase preconceptions about students derived by studying the average variation or average variations, analysis functions and their properties and some asides about limits and continuity. A second phase involves design an educational strategy to influence the improvement of academic performance, while also can strengthen learning concepts to reach the understanding of the same by contextualized problems and other applications to everyday life, thus seeks to promote creativity in students and a third stage correlated variables that will produce strengths in methodology and rationale regarding the identification of their own preconceptions, progressive mastery of definitions, concepts and situations in the applied the concept of derivative.

To investigate how it influences the teaching strategy teaching the concept of derivative and its applications a descriptive study comparing the academic performance between a control group with traditional methodology and an experimental group to which was applied a methodology based on creative learning set, supported by the use of technological resources, one group experimental to which is applied the strategy teaching practices based on creative learning, and a control, which served as a reference point to know the variations resulting: for the design were two groups the strategy applied in teaching mechanical physics. The starting point of the research proposal, tends to delimit the population or group of entities participating in a study with certain characteristics which was constituted by 579 students in grade eleven 128 of the average technique. Similarly, the end of the course Post-test, for which a number of problems for each competition (reasoning, communication and resolution) was employed and applied.

The course was considered as a sign [3], according to regulations, four-hour (55 minutes each class a week). Three correspond to the appropriation of concepts and practice supported the experimental use of digital technologies. Furthermore, two complementary devoted to advice, which are made complementary activities hours. With regard to evaluation, the institutional system of student Assessment indicates at least 5 tests with a value of 50%, corresponding to reports and results of practices with a percentage of 30% and an exam worth 40% note and a final test where the units are evaluated views to the filing dates set by the academic calendar. The structure of the teaching strategy notes stages of exploration: to make known the content, units and unit objectives, evaluation criteria and reference sources. This is done at the beginning of the period in the classroom; structuring, correspond to problems posed and solved in the classroom where students identify which concepts and principles that can take me to propose a possible solution by applying creativity being the teacher who guides you through the resolution process are. Guided troubleshooting activities are conducted in class time, they are solved with the support of teachers and through collaborative work. Concept maps are used to see the creativity regarding new concepts. Each student must do so personally; they are made at the end of each unit. Complementary activities supported practices in the use of technological tools, from each subject a practical activity guided, which is put into practice the knowledge acquired is made. In the pre-test and post-test, they are open try-outs on preconceptions associated with the theme of each unit are performed at the beginning and end of the proposal. Activities developing a fitness creativity: it consists of the production of brochures, maps and conceptual models that support the appropriation of concepts.

The transfer is the space that each group, experimental and control was provided so that through collaborative work share the results of methods and techniques applied to reach the solution.

The H_1 hypothesis was accepted that the variances are equal if ρ (value obtained) is (hypothesis researcher). The H_2 hypothesis there is a difference between the variances, the test of Kolmogorov-Smirnov KS for large samples or the Shapiro Wilk when the sample size is $n < 30$ [4]. Hypothesis H_1, for a confidence level of 95% the value of significance is the data come from a normal distribution, ρ itself (value obtained) is (hypothesis researcher). $\rho \geq \alpha\ \rho < \alpha\ \alpha = 0.05\ \rho \geq \alpha$. The hypothesis H_2 data do not come from a normal distribution $\rho < \alpha$. To analyse the value of t-student test (nonparametric). Hypothesis H_1 for a confidence level of 95% the value of t must be between [5], there is no conceptual difference between the groups at baseline experiment (hypothesis researcher) $-1.7 < t < 1.7$ [4]. The H_2 hypothesis if the value of t is not within these ranges of values if no conceptual difference between these two groups, starting the experiment. The U-Mann-Whitney (non-parametric) for independent samples to confirm the homogeneity of concepts of experimental and control groups at the beginning of the experiment, if not defined the results in the Student t test [6] there is no conceptual difference between the groups at baseline experiment $-1{,}96 < z < 1.96$ [7]. Design a teaching strategy based on meaningful learning in the teaching of the derivative to improve the academic performance of students follows the following methodology: the student test (parametric) for independent samples to analyze the performance of the experimental and control groups in the test post-test must meet normal frequency distribution and equal variances $-1.7 < t < 1.7$ [8]. The instrument used were test pre-test and post-test examples of conceptual applications, which they assessed as follows, pre-test of 21 multiple-choice questions with a single answer on the concepts of functions and properties, equation and slope of the line, the limits of functions and continuity. Post-test questions 13, 10 with multiple selection only answer and three open questions include concepts and principles of application of the derivative.

Cronbach alpha coefficient for the 21 items of pre-test is a value of 0.792 enabling accept the reliability of the test [9]. Through this study questions the consistency confirmed, since they must answer a clear intent and purpose. The reliability value is 0.7677, which provides the first test of an acceptable reliability, although the need for modifications thereof evidenced. Output test (post-test) consists of 10 multiple-choice questions and three of solving problems involve the development of mathematical reasoning and communication skills. The instrument was applied to 150 students who were part of the control and experimental groups; a Cronbach's alpha of 0.704 was obtained.

3. Results

Pre-test results demonstrate the difficulties students have in mastering and are required to arrive at an advanced or satisfactory understanding of the concept of derivative level. The 32% of students do not identify functions, do not handle the concept and know their characteristics and properties, the 46% identify the graph of a function, but have difficulty analyzing and interpreting the behavior of the curves, 12% recognizes and identifies the characteristics and properties of functions, but does not understand their applications and only 10% sample the ability to understand the function concept and its application to modelling phenomena context [8]. On the other hand, 24% does not include the concept of slope as a ratio between two quantities, 50% calculates slope but neither interprets its meaning, 16% performed their calculation and interprets its meaning and finally a 10% slope understand the concept and interprets its meaning in any everyday situation. As for the equation of the line for 34% of the evaluated not recognize relationship between the variables, does not determine the algebraic expression and does not apply to model real situations, 64% perform certain calculations of the equation and only 2%, it recognizes as a tool for modelling real situations. The results made it possible to design an educational strategy to support ownership and understanding of the concept of the derivative. Comprehension levels consist of a qualitative description of student performance.

Level 4 (Advanced), 14% of those tested fails to understand the concept of average rate of change (medium speed), 3% instantaneous speed, 3% the concept of derivative (as slope of the tangent line or and limit quotient), 3% and communicate achievement solve mathematically by applying techniques and referral criteria and 3% achieved by proposing reasoning and demonstration; which it allows us to

argue that found students who have the ability to deduct and combine procedures to perform the tasks requested; level 3 (satisfactory), 45% of those tested fails to understand the concept of average change rate (average rate), 59% instantaneous speed, 66% the concept of derivative (as slope of the tangent line or limit the incremental ratio), 55% achievement resolve and communicate mathematically by applying techniques and criteria for referral and 41% achieved reasoning by proposing and demonstration, i.e. students analyzed procedures to develop in the best way the requested task; at level 2 (minimum) 31% of the evaluated shows difficulty in trying to understand the concept of average rate of change (average speed), 31% instantaneous speed, 14% the concept of derivative (such as pending tangent or limit quotient) straight, 21% failed to resolve and communicate mathematically by applying techniques and criteria for referral and 14% does not reach a reasoning by proposing and visual demonstration of the proposed problem; which allows us to establish that a certain percentage of students have difficulty developing skills to deduct and combine procedures to perform the tasks requested.

It evidenced one differentiation capacity possible procedures to solve very simple problems; while in level 1 (insufficient), 7% of those tested fails to understand the concept of average change rate (average rate), 10% has trouble visually interpret the instantaneous speed, 10% fails to understand the concept of derivative (as slope of the tangent line or limit the incremental ratio), 28% are not resolved and or communicate mathematically concrete results by applying techniques and criteria for referral and 34 % shows a marked difficulty in finding arguments by proposing and demonstration, which allows us to demonstrate that a high percentage of students are limited to identifying the tasks demanded, leaving aside the ability to establish criteria that allow him to argue solutions by synthesizing graphical and analytical information arising in activities classroom, 28% are not resolved and no concrete results communicate mathematically by applying techniques and criteria for referral and 34% shows a marked difficulty in finding reasoning by proposing and demonstration; which allows us to demonstrate that a high percentage of students are limited to identifying the tasks demanded, leaving aside the ability to establish criteria that allow him to argue solutions by synthesizing graphical and analytical information arising in activities classroom, 28% are not resolved and no concrete results communicate mathematically by applying techniques and criteria for referral and 34% shows a marked difficulty in finding reasoning by proposing and demonstration; which allows us to demonstrate that a high percentage of students are limited to identifying the tasks demanded, leaving aside the ability to establish criteria that allow him to argue solutions by synthesizing graphical and analytical information arising in activities classroom.

The control group had a higher concentration of students in insufficient and minimum levels confirming that the traditional teaching methods as building graphics, the layout of the tangent line and the display changes or incremental ratio, including the limit; hinder the achievement of the objectives. About 90% of students in this group were limited to performing procedures mechanically involving mastery of certain techniques bypass therefore showed difficulties when modelling problem situations or reason situations requiring propose and argue recognizing that no it is so easy to assimilate and rebuild from the concept of limit or slope of the tangent to the curve straight.

The results show that there are no significant early in the course $p > 0$ value (the null hypothesis of equal averages is accepted) difference [10]. On the other hand, significant differences between control and experimental $p < 0$ (different average) it is observed. Finally, it is evident that the experimental group experienced an improvement in the results of the test averages. These results are significant for the statistical throw an acceptance of the initial hypothesis [11]. Levene statistic takes a value small enough to not reject the hypothesis of homoscedasticity usual levels of significance [12]. The value of the test Fisher for pre-test 0.891 with significance level equal to 0.350, significantly different from 0 therefore the null hypothesis of equal means is accepted. According to the results of the interview applied to the three teachers in the area of mathematics that are part of the educational institution José Maria Córdoba, it is evident that they have the same conception of the derivative and its importance in the process of teaching and learning. As the use of new communication technologies as enabling tools. It is important to note that when analyzing levels of understanding of the concept of derivative students placed in the "insufficient" level did not show the ability to explain the concept of derivative and its

applications to solving problems [13]. Barely showed mastery of some technical and properties to solve exercises mechanically. In the second, minimum level managed to establish a relationship between the basic concepts rate of change and incremental ratio. Which shape the derivative, but in them some difficulty reason. analyze and interpret a progressive construction scheme concept persist derivative. At the third level, satisfactory managed to produce verbalizations related to cognition.

4. Conclusions

Creating playful strategy allowed improving levels of compression and enrich the knowledge of the use and educational use of information and communications technology in the development of activities that produce significant learning of the concept of derivative, overcoming obstacles presented in the classroom when the appropriation of new knowledge was sought. The use of GeoGebra software and other applications is beneficial because the classroom environment, in turn constitutes a working tool to capture the attention of students, making them more participatory, creative and proactive, improved the process of teaching and learning. Students showed great interest, although different learning rates was perceived. Interpretation and analysis of the results showed the difficulty students in the control group in the domain of knowledge previous (functions, properties and characteristics of functions, limits, continuity), in addition to the construction and appropriation the concept of derivative as the limit of incremental ratio or slope of the tangent to the curve at a given point, which becomes even more complex by the symbols used. By contrast students who took part in the experimental group achieved through visualization and modelling situations contextualized understanding of the concept of derivative from a more analytical and interpretive perspective.

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